A Simple Method for the Design of Hybrid Electric Power-Split CVTs: A Case Study

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Abstract. A brief parametric method for the preliminary design of hybrid electric Power-Split CVTs is performed. The desired torque-speed characteristic is obtained, and general control and optimization criteria are introduced in order to minimize the size of the required electric machines. The optimization is performed by mean of scheme-independent functional parameters, while a specific constructive design is proposed only later, along with possible full electric operations.

Keywords: Power-Split CVTs · Hybrid electric transmission Parametric design · Downsizing

1 Introduction

A discrete planetary gearbox is supposed to fulfill basic functioning requirements; notably, the maximum speed and the overdrive conditions determine the selection of the highest gears, while the lowest gear is the result of acceleration and slope climbing criteria. The total number of gears and their relative gear staging is a compromise between driving pleasure, engine's efficiency, weights, costs and space requirements [1, 2].

Unfortunately, Power-Split CVTs are subject to mutable torque and speed ratio boundaries and show fluctuating efficiencies. This is due to the constructive limits of the variator drive and to its ability to strongly interfere (also actively) with the overall power flow distribution. Assessing the mechanical losses is troublesome as well, as the gears' loss factors are subject to numerous discontinuities [3]. In other terms, Power-Split CVTs lack fully definite design reference points. As a result, the design problem is usually addressed by mean of an explorative approach, generating a large discrete domain of possible layouts with constructive ratios varying continuously within bounded intervals. Indeed, the architecture, component parameters, and control strategies mutually interfere with each other, thus affecting vehicles' performances, and certainly it will exist a specific architecture (and related constructive parameters) that

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optimize the actual performances of the HEV for a certain duty cycle, for instance in terms of fuel economy and acceleration capacity [4, 5].

Nonetheless, the preliminary design of any PS-CVT and the formulation of a primitive control strategy can be performed without assuming a specific driveline layout, as generic torque and speed requirements can be addressed by mean of a parametric approach [3, 6–8], as it will be illustrated below by mean of a practical example. In particular, it is not necessary to specify the parameters of the vehicle as well; on the contrary, the method provides general design and control guidelines that can be easily applied to a range of possible different scenarios, thus constituting an excellent starting point for designers operating in any field. Moreover, the approach makes easier the individuation of the whole sub-domain of functionally equivalent solutions, making any explorative approaches both faster (smaller domain, better starting points) and more reliable.

The paper is structured as follows: a general model for PS-CVTs is presented, and the control strategy for the minimization of the power flowing through the electric path is described; the design requirements, in terms of output torque-speed characteristic, are defined; the optimal functional parameters (mechanical points) are obtained; the synchronous ratio is chosen; a functional layout is proposed; full electric modes are discussed.

2 PS-CVT Parametric Model

A Power-Split CVT (PS-CVT) is a driveline including a continuously variable unit (CVU), and a Power Split Unit (PSU). In hybrid electric PS-CVTs, the CVU is the ensemble of electric motors, batteries and required power electronics, while the PSU is any planetary transmission able to redistribute the input power.

The indexes "*in*" and "*out*" refer to the input and output shaft of the PS-CVT, while "*i*" and "*o*" are the input and output shafts of the CVU. The mechanical points $\tau_{\#i}= (\omega_{out}/\omega_{in})|_{\omega_i=0}$ and $\tau_{\#o}= (\omega_{out}/\omega_{in})|_{\omega_o=0}$ (overall speed ratios for which either "*i*" or "*o*" is still) and their related CVU speed ratios $\tau_{o\#i} = (\omega_o/\omega_{in})|_{\tau_{\#i}}$ and $\tau_{i\#o} = (\omega_i/\omega_{in})|_{\tau_{\#o}}$ determine the relationships between powers, torques and speeds of the main shafts as functions of the overall speed ratio τ and torque ratio Θ, whichever it is the PS-CVT (see Eqs. (1)–(9) [7, 8]). Speed relationships:

$$\tau = \frac{\omega_{out}}{\omega_{in}} \tag{1}$$

$$\frac{\omega_i}{\omega_{out}} = \tau_{i_{\#o}} \frac{1 - \tau_{\#i}/\tau}{\tau_{\#o} - \tau_{\#i}} \tag{2}$$

$$\frac{\omega_o}{\omega_{out}} = \tau_{o_{\#i}} \frac{1 - \tau_{\#o}/\tau}{\tau_{\#i} - \tau_{\#o}} \tag{3}$$

Torque relationships:

$$\Theta = \frac{T_{out}}{T_{in}} \tag{4}$$

$$\frac{T_i}{T_{out}} = -\frac{\left(1/\Theta + \tau_{\#o}\right)}{\tau_{i_{\#o}}} \tag{5}$$

$$\frac{T_o}{T_{out}} = -\frac{\left(1/\Theta + \tau_{\#i}\right)}{\tau_{o_{\#i}}} \tag{6}$$

Power relationships:

$$\eta = -\frac{P_{out}}{P_{in}} = -\tau\,\Theta \tag{7}$$

$$\frac{P_i}{P_{out}} = \frac{\left(1 - \tau_{\#i}/\tau\right)\left(1/\Theta + \tau_{\#o}\right)}{\tau_{\#i} - \tau_{\#o}} \tag{8}$$

$$\frac{P_o}{P_{out}} = \frac{\left(1 - \tau_{\#o}/\tau\right)\left(1/\Theta + \tau_{\#i}\right)}{\tau_{\#o} - \tau_{\#i}} \tag{9}$$

In Eqs. (4)–(9) the mechanical losses in the PSU are neglected; powers are assumed positive if entering the PSU, and since the CVU is subject to several power flow reversals, its shafts "*i*" and "*o*" can be assigned arbitrarily, for example assuming $0 < \tau_{\#i} < \tau_{\#o}$. The optimization requires to select the best pair of mechanical points and a proper control strategy in order to minimize the size of the electric machines, in terms of overall speed ratio τ and torque ratio Θ. In particular, the conditions that make equal and opposite P_i and P_o are:

$$P_i = -P_o = \frac{\left(\tau - \tau_{\#i}\right)\left(\tau - \tau_{\#o}\right)}{\tau_{\#i} - \tau_{\#o}} P_{out}; \tag{10}$$

$$\Theta = -1/\tau \tag{11}$$

While P_i and P_o are identical if:

$$P_i = P_o = \frac{\left(\tau - \tau_{\#i}\right)\left(\tau - \tau_{\#o}\right)}{\tau_{\#i} + \tau_{\#o} - 2\tau} P_{out}; \tag{12}$$

$$\Theta = \frac{\left(\tau_{\#i} + \tau_{\#o}\right) - 2\tau}{\tau\left(\tau_{\#i} + \tau_{\#o}\right) - 2\tau_{\#i}\tau_{\#o}} \tag{13}$$

Equations (10) and (11) imply that the engine is delivering the required output power, while Eqs. (12) and (13) imply either a deficit or a surplus. In particular, the electrical power calculated by Eq. (10) is smaller than that calculated by Eq. (12) within mechanical points. According to the previous criteria, the optimal value of the overall torque ratio Θ is always within the range $-1/\max(\tau_{\#i}, \tau_{\#o}) > \Theta > -1/\min(\tau_{\#i}, \tau_{\#o})$, and in such conditions the engine will deliver more power ($\tau < \min(\tau_{\#i}, \tau_{\#o})$) or less power ($\tau > \max(\tau_{\#i}, \tau_{\#o})$) than necessary when working outside the mechanical points.

Eventually, if $\Theta > -1/\max(\tau_{\#i}, \tau_{\#o})$, it is always possible (and beneficial) to choke the engine in order to meet Eq. (11) or (13). Vice versa, if it is $\Theta < -1/\min(\tau_{\#i}, \tau_{\#o})$ and the engine is at WOT (because of the high power demand), choking the engine will always worsen the MGs' operating conditions, because raising Θ implies both higher P_i and P_o.

3 Design Requirements

Our goal is to design a PS-CVT able to mimic the behavior of a full electric vehicle in terms of output torque.

In our example, the available internal combustion engine delivers its maximum torque of 170 Nm at 2500 rpm and its maximum power of 80 kW at 5500 rpm. In order to facilitate the reproducibility of the results, the previous data is used to interpolate its WOT torque function by mean of a third grade polynomial (see Fig. 1); nonetheless, experimental data can be used as well. The engine operative speed range is between 800 rpm and 5800 rpm.

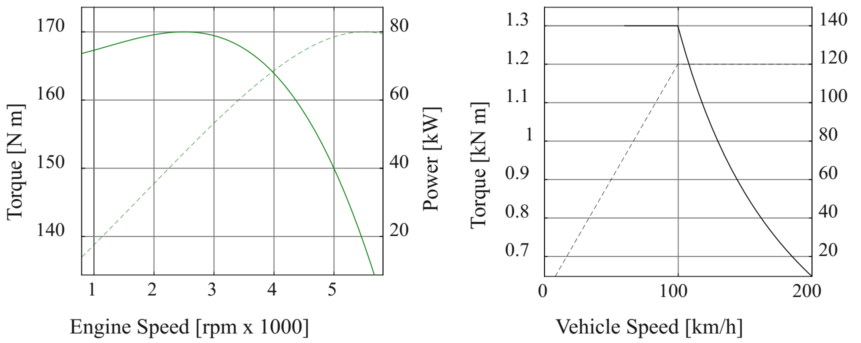

Fig. 1. Left figure: WOT torque and power (dashed green line) for the available I.C.E. Right figure: desired torque and power (dashed black line) at wheels. (Color figure online)

The vehicle must be able to deliver its maximum torque until it reaches the speed of about 100 km/h (1300 Nm in the speed range between 0 and 100 km/h), and then it must provide at least the same amount of output power until it reaches the maximum speed of 200 km/h (120 kW between 100 and 200 km/h, see Fig. 1).

These performances require the assistance of the electric motors, so the charge condition of the batteries is assumed to be adequate. In particular, the engine alone can deliver up to 80 kW, so it is not able to provide enough power starting from 67 km/h (see Fig. 1). The electric motors can deliver their (maximum) constant torque up to their base speed of 3500 rpm, and then their (maximum) constant power up to 10000 rpm. As usual for PS-CVTs, they will be used also to perform full electric operations, regenerative breaking, cranking of the engine and general speed control.

The main optimization objective is to minimize the size of the electric motors and, consequently, of the related power electronics. Since the vehicle will be equipped with a couple of electric motors/generators (MGs), it is appropriate to provide at least for a full electric vehicle (FEV) functioning within urban areas. Accordingly, alternative functioning modes for low speeds will be addressed in this paper.

4 Optimization of the Functional Parameters

The optimization, which changes the mechanical points and the local torque ratio Θ until the least maximum electrical power is obtained, converged to an output-split solution, as for a wheel radius $R = 0.3\,\mathrm{m}$, the optimal mechanical points are $\tau_{\#i} \rightarrow \infty$ and $\tau_{\#o} = 0.218$. For most of the vehicle's speed range, the overall torque ratio Θ follows the condition (13) that makes equal the power supplied by the MGs.

The engine is always operated at WOT, and each MG must deliver about 34.3 kW. Interestingly, the speed ratio and the torque ratio are almost constant ($\tau = 0.305$, $\Theta = -7.72$) for speeds up to 100 km/h, then τ changes very slowly until the engine reaches its top speed (at about 172 km/h), beyond which Eq. (13) cannot be fulfilled anymore. As a result, P_i and P_o diverge, and the latter takes again its peak value at the top speed of the vehicle, for $\tau = \frac{25}{3\pi}\frac{V_{max}}{n_{max}R} = 0.305$ and $\Theta = \frac{P_{out}(V_{max})}{\tau P_{in}(n_{max})} = -4.96$ (see Fig. 2).

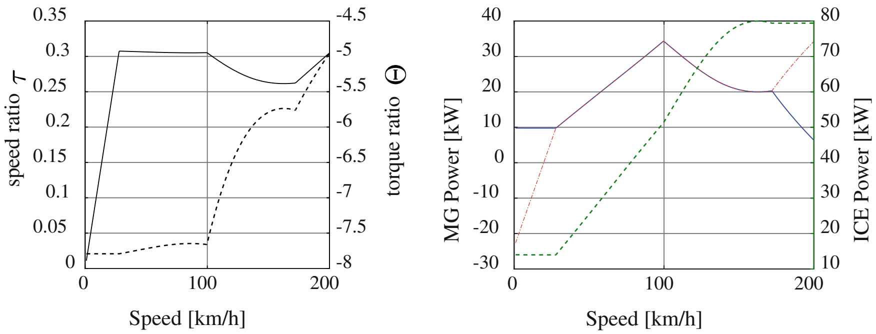

Fig. 2. Left figure: Optimal overall speed and torque (dashed line) ratios for the PS-CVT. Right figure: powers supplied by motors (blue for P_i, dash-dot red for P_o) and engine (dashed green). P_i and P_o overlap between 27 km/h and 172 km/h. (Color figure online)

Basically, the optimization makes equal and minimize the peak electrical power for the critical conditions, which are the base speed of 100 km/h (the vehicle must provide both the maximum torque and power) and the top speed (P_o reaches its peak again). Unfortunately, the downsizing of the engine forces the control strategy to resort to

Eq. (13) instead of Eq. (11) between mechanical points, while the engine's speed limits cause the divergence between electrical powers for speed lower than 27 km/h and higher than 172 km/h.

It is worth noting that between 100 and 200 km/h the electric motors could deliver more power than necessary, so in this speed range the real performances can exceed the design requirements.

4.1 Synchronous Ratio

Both the MGs have to be faster than their base speed at 100 km/h (first power peak), and slower than their maximum speed in the whole operation range. At 100 km/h the engine is rotating at about 2900 rpm (see Fig. 3). In particular, assuming that the base speed of both the MGs is higher (3500 rpm) and choosing $\tau_* = 0.305$ (see Fig. 2) as the synchronous ratio, an additional gear stage $k_{in} = 0.82$ linking the engine to the input shaft is required in order to make the MGs reach their base speed at 100 km/h; accordingly, the final drive ratio is $k_{out} = \tau_* k_{in} = 0.25$, and since the additional multiplication gear stages are on main shafts, the relative CVU speed ratios (see Eqs. (2)–(3)) are ($\tau_{\#i} \rightarrow \infty$ and $\tau_{\#o} = 0.218$):

$$\tau_{i_{\#o}} = \frac{1}{k_{in}} \frac{\tau_{\#o} - \tau_{\#i}}{\tau_* - \tau_{\#i}} = 1.22; \tag{14}$$

$$\tau_{o\#i} = \frac{1}{k_{in}} \frac{\tau_{\#i} - \tau_{\#o}}{\tau_* - \tau_{\#o}} = 14.0 \cdot \infty \tag{15}$$

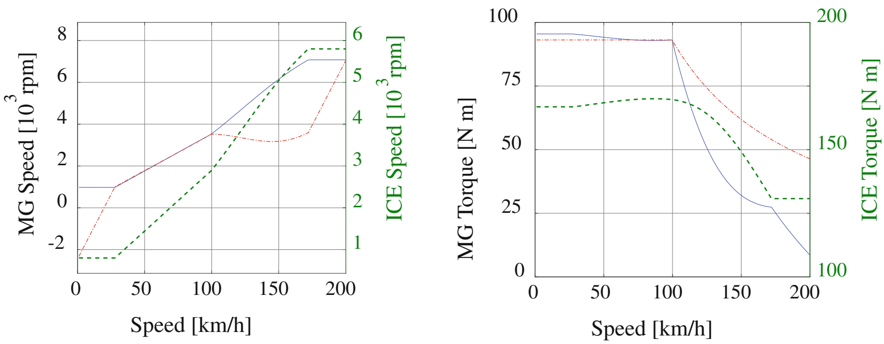

Fig. 3. Overall speeds (left figure) and torques (right figure) for the PS-CVT operating with the overall torque and speed ratios of Fig. 2 (blue for MG_i, dash-dot red for MG_o, dashed green for the ICE) (Color figure online)

The functioning of the PS-CVT is now totally defined and absolute torques and speeds (see Fig. 3) can be calculated thanks to Eqs. (2)−(3).

The speed of 2900 rpm would be the upper limit for the base speed of the MGs in order to avoid the additional multiplication gear stage k_{in}. In theory, eliminating k_{in} would be beneficial for overall efficiency, encumbrances and costs. Yet, since the base speed of the available MGs is higher (3500 rpm), in this circumstance significantly bigger machines would be necessary, because the electric motors would be supposed to deliver more torque than about 95 Nm between zero and 100 km/h. In particular, they should be able to deliver about 43 kW each, thus being oversized in respect of the required maximum power, which is inferior to 35 kW.

5 Functional Layout

The PS-CVT layout can be identified thanks to the design chart [7]. The characteristic curve that offers constructive ratios within −1/3 and −2/3 for synchronous ratios around 0.3 is $\phi_{in/o}^{out}$ (see Fig. 4), and its value, calculated for $\tau_* = 0.305$ ($\tau_{\#out} = 0$ and $\tau_{\#in} = \infty$ by definition) is [8]:

$$\psi_{R/S}^{C} = -\frac{z_S}{z_R} = \phi_{in/o}^{out}(\tau_*) = \frac{\tau_* - \tau_{\#o}}{\tau_* - \tau_{\#in}}\frac{\tau_{\#out} - \tau_{\#in}}{\tau_{\#out} - \tau_{\#o}} = -0.4 \qquad (16)$$

$\psi_{R/S}^{C}$ is known as the Willis' ratio of the planetary gear train. According to the position of the indexes, the engine is linked to the ring gear (and MG_i) by mean of the fixed-ratio k_{in}, while MG_o is linked to the sun gear; the output shaft links the wheels to the planet-carrier by mean of the final drive k_{out} (see Fig. 4).

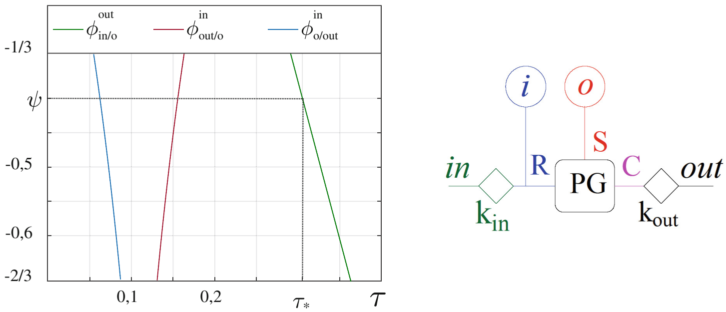

Fig. 4. Design chart for $\tau_{\#i} = -\infty$ and $\tau_{\#o} = 0.218$ and functional PS-CVT layout for $\tau_* = 0.305$ and $\psi = -z_S/z_R = -0.4, k_{in} = \omega_{in}/\omega_R = 0.82, k_{out} = \omega_{out}/\omega_C = 0.25$.

In the functional layout depicted in Fig. 4, the square represent a PG, while R, C, S denote the shafts linking respectively its ring, carrier and sun gear. Rhombi represent fixed gear ratios, and circles electric machines.

5.1 FEV Modes

The electric machines have a maximum torque of 95.5 Nm each and a combined power of 70 kW, which should be enough to guarantee the required torque of 1300 Nm (see Fig. 1) up to 58 km/h, i.e. within urban areas.

A full electric mode E1 can be achieved by disengaging the engine from the input shaft by mean of a clutch 1 (see Fig. 5 and Fig. 6), albeit such mode is not able to comply with the maximum torque requirements, as MG_o cannot deliver its maximum torque. The reason is that the latter is proportional to the torque applied to the input shaft, which is now operated by MG_i alone, without the contribution of the engine. As a result, the E1 mode is able to deliver just 535 Nm until the overall output power reaches 70 kW at about 140 km/h. This condition takes place when MG_i is running at its base speed of 3500 rpm and MG_o spins at 8750 rpm. The same applies to reverse vehicle's speeds, provided that the motors spin in the opposite directions. On the bright side, E1 permits to regulate the speed of the electric machines, letting them work towards the best average efficiency. It is possible to start the engine at any moment provided that the speed of MG_i is higher than the idling speed of the former. A synchronous cranking of the engine can be accomplished if MG_i has been kept still, which limits the concurrent maximum vehicle's speed to about 80 km/h.

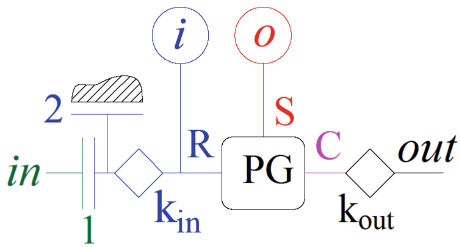

Fig. 5. Modified functional scheme for the PS-CVT with $\tau_{\#i} = -\infty$ and $\tau_{\#o} = 0.218$, $\tau_{*} = 0.305$, $\tau_{i\#o} = 1.22$ and $\tau_{o\#i} = 14.0\infty$ with switching clutches.

In conclusion, mode E1 cannot accomplish the high performances specified in Fig. 1. Such performances might be achieved without sophisticating the design concept by changing the final drive ratio to a steep $k_{out} = 0.103$ (and thus it would be also $k_{in} = 0.338$), but this would require MGs able to spin at over 17000 rpm in PS-CVT mode (at 200 km/h) and possibly one additional reduction stage in order to realize k_{out} itself. Accordingly, the constructive parameters will stay unaltered and the full electric mode E1 will be restricted to general part-load operations, which constitute the most part of common driving cycles. It is worth noting that the proposed sign of k_{in} and k_{out} is not binding, as what matters for the motors are their absolute speeds, so either can be changed for constructive reasons.

A different solution E2 requires to block the input shaft altogether by mean of a brake 2 (see Figs. 5 and 6), so that MG_o could deliver its maximum torque. In this condition the speed of MG_o is 14 times the speed of the wheels, and thus the output

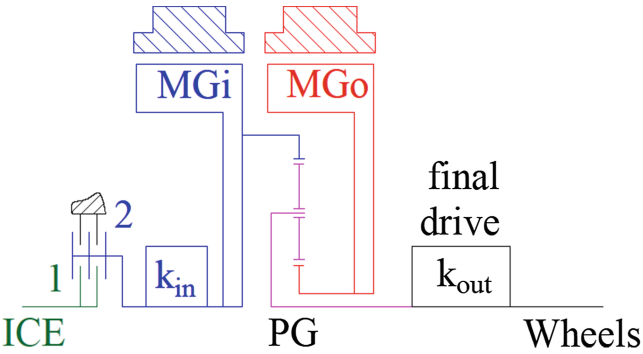

Fig. 6. Proposed constructive scheme for the PS-CVT with $k_{in} = 0.82$, $k_{out} = 0.25$, $\psi = -0.4$. (Color figure online)

torque can be even slightly higher than required (1340 Nm). Obviously, this result is limited to a narrow speed range across the neutral gear (within 29 km/h), and MG_o is going to over-speed at about 80 km/h. Yet, starting from 70 km/h the mode E1 offers better performances, so a switch is supposed to take place concurrently. Such switch is necessary also in order to crank the engine, which means that, before performing it, adapting the output torque to the E1 levels is necessary. Such operation may be perceived as particularly uncomfortable, as the torque jump can be consistent especially at low speeds.

Definitively, E2 can provide the necessary torque for uphill driving, but cannot guarantee acceleration performances comparable to the output-split mode. On the other hand, the output-split is capable of delivering slightly more than the prevented 1300 Nm in the $0 - 100$ km/h range, and up to 150 kW at about 175 km/h.

In Fig. 6 the engine $(in = ICE)$ is linked to the electric motor MG_i $(i = MG_i)$ by mean of the clutch 1 (green-blue) and the gearbox k_{in}. The planet carrier (purple) of the PG is linked to the wheels $(out = wheels)$ by mean of the final drive k_{out}. MG_i spins with the ring gear (blue) of PG, while MG_o spins with the sun gear (red). If the clutch 2 (blue-black) is engaged, the FEV mode E2 occurs.

6 Conclusions

In this paper, the design of a PS-CVT has been performed starting from generic torque and speed requirements, with the objective to minimize the size of the necessary electric motors. In particular, the control strategy equalizes the electric powers' magnitude whenever it can, with absolute results depending on the optimization of the functional parameters.

The choice of the synchronous ratio, and thus the selection of the constructive ratios of the transmission, has been performed considering the operative limits of typical electric machines. The engine start-up and the reverse speed operation are guaranteed, and additional full electric functioning modes are proposed.

The paper shows that a heuristic or explorative approach in not always necessary in order to perform the optimization of a PS-CVT. On the contrary, it is possible to limit the subsequent layout exploration to the solutions that are already known to reach optimal results.

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