Mechanism and Machine Theory 116 (2017) 294–309

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Functional design of power-split CVTs: An uncoupled hierarchical optimized model

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ARTICLE INFO

Article history:
Received 17 March 2017
Revised 12 May 2017
Accepted 5 June 2017

Keywords:
Continuously variable transmission
Power-split CVT
Functional design
Configuration design
e-CVT
Hybrid

ABSTRACT

This paper provides a new model for the preliminary design of compound power-split CVTs.

Unlike the existing models, the presented method allows the engineers to prioritize functionality and efficiency of the transmission, while delaying the choice of the involved gear sets' layout as long as possible. The design approach follows a specific priority order, and each step deals with one particular issue, without mutual interference.

A smart *design-chart* eases the assessment and the comparison of the only eligible alternatives, and eventually leads to a final feasible constructive scheme, which can be an excellent concept for further optimization and implementation.

Moreover, the model is general and straightforward: it does not resort to arbitrary or practice dictated choices, it does not require convoluted and laborious numerical procedures, such as large systems of equations, inequalities or iterative computations.

1. Introduction

In recent years, increasing powertrain efficiency has become a significant issue. In this scenario, the ability to continuously change the speed ratio between the input shaft and the final drive, so as to optimize the performances of the main engine, continues to be one of the most interesting and promising subjects, especially in the automotive field. However, the use of mechanical, electrical, or hydraulic variators still entails several drawbacks, mostly in terms of transmissible power, width of the speed range, efficiency, and costs [1–10]. A very effective solution for step-less transmissions, proposed by several authors and producers since the second half of the last century, involves splitting the primary mechanical power into two paths, and trying to convey the major part through ordinary and planetary gearboxes. Such transmissions exhibit interesting characteristics and performances, which make them suitable for earthmovers and hybrid vehicles. Indeed, its recent application in hybrid electric cars is intended to enhance both vehicle dynamics and energy recovery systems, and it offers promising prospects of cost reduction, despite the increased constructive complexity, due to the possible downsizing of both electric and thermic propulsion systems [11–15].

In comparison with the variator alone, the adoption of a single-mode power-split continuously variable transmission (PS-CVT) can amplify either the power class or the speed ratio range of the driveline itself, but not both. Indeed, the possible onset of power recirculation phenomena, if it is not properly mitigated, can seriously damage the overall efficiency. However, the previous positive results can be both obtained by using multi-mode PS-CVTs, i.e. transmissions that can perform two or more contiguous speed ranges, which commute to each other by brakes or clutches [16].

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http://dx.doi.org/10.1016/j.mechmachtheory.2017.06.003

Nomenclature

The apostrophe denotes a variable normalized with respect to its initial value, which is marked with the subscript $(.)_m$ in order to point out its concurrency with the overall initial transmission ratio τ_m; similarly, we mark its end value with the subscript $(\cdot)_M$ in order to point out its concurrency with the overall final transmission ratio τ_M.

An over-lined power or torque symbol refers to real working conditions.

CVU	continuously variable unit
IN	input of the transmission
OUT	output of the transmission
i	input of the CVU
o	output of the CVU
j	jth shaft
τ	overall speed ratio
τ_j	dimensionless angular speed (i.e. speed ratio) of the jth shaft
τ_v	CVU speed ratio
A	overall ratio spread
A_j	ratio spread of the jth shaft
A_v	CVU's ratio spread
P_j	ideal power transmitted by the jth shaft
p_j	dimensionless ideal power transmitted by the jth shaft
p_i	dimensionless ideal power transmitted by the CVU
η_v	apparent efficiency of the CVU
η	apparent efficiency of the PS-CVT
T_j	ideal torque applied to the jth shaft
θ_i	dimensionless ideal torque applied to the input of the CVU
$\psi^{z}_{y/x}$	fixed-z speed ratio of the planetary gear train in a three port differential
k_x	fixed speed ratio on the xth shaft of a three port differential
τ_*	overall speed ratio while a planetary gear train is synchronous
$\phi^{z}_{y/x}$	generic three-port differential characteristic function
ω_j	angular speed of the jth shaft
α_j	implicit functional parameter
$\tau_\#$	optimized overall transmission ratio
$\tau_{\#,j}$	overall transmission ratio when $\omega_j = 0$

To date, numerous authors have proposed suitable methods for the study of similar transmissions.

Great attention has been paid to the analysis of power flows and losses, and many studies have been based, or independently accomplished [17–22], on the pioneering works of Macmillan [23,24], French [25], Sanger [26] and Polder [27,28]. Nevertheless, a vast literature, both experimental and theoretical, deals with the analysis, optimization and control of specific constructive solutions. Numerous papers are very well detailed (see [29–42]) and they manage to describe some of the key features of specific case studies, but are lacking in generality.

Nonetheless, to the best of our knowledge, a really general, simple and concise analytical approach to the preliminary design problem is still lacking. Indeed, to design PS-CVTs can be very tricky because of their constructive complexity and variety, and, despite the time and resources deployed, arbitrary or practice dictated choices could lead to sub-optimal solutions. Accordingly, a simple tool, to be promptly used in order to assess their feasibility and potential effectiveness, is required.

Contrariwise, most of the universal design methods that have been proposed involve a large amount of variables without a direct or evident physical meaning, requiring a deep knowledge of the theory for their proper utilization. Usually, they are intended to deal with compound planetary transmissions with a discrete number of ratios, but might be adapted to address PS-CVTs. For instance, in [43,44], Mathis and Rèmond provided a fundamental analytical expression (primitive parametric kernel), which can be adapted to address, through Boolean parameters, the fundamental physical variables (such as speed, torques and efficiency) of any compound discrete planetary transmission.

Similarly, graph theory [45–50] appears well suited for the analysis and design of such transmissions, and it is widely used to explore the possible solutions; yet, identifying the most suitable layouts, among the feasible ones, is not immediate. For instance, in [50] a configuration synthesis method, specifically conceived for series-parallel transmissions, is presented: it is based on the topological characteristics of planetary gear trains and of some existing designs, enlisting all the analogous solutions. However, there is no way to predict which could be the best solution, unless simulating them all.

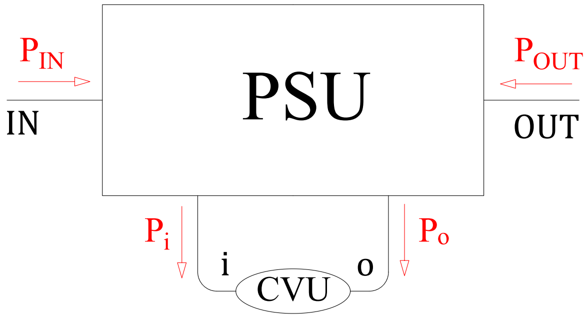

Fig. 1. Basic layout of a power-split CVT, which is made up of a PSU and a CVU. The represented power flows are assumed positive.

For the same reasons, numeric or graphic design methods that require presuming the arrangement of the driveline, lead to too specific, and potentially suboptimal, results.

For example, the "lever analogy" is widely used by researcher, as it consents a swift qualitative analysis of the operation of a simple known compound planetary gear system. Usually, it is applied if ordinary gear trains are not involved. It is used in [51], for the design of a GM AHS-2 transmission, whose constructive parameters are the result of the optimization of a specific drive cycle, as well as in [52] and [53]. Yet, they deal specifically with HEVs and presume the existence of two mechanical points. In the latter, the layout is the result of the electric motors' dimensioning, but the screening process remains quite laborious.

In [54] an exhaustive numerical approach to the design problem is presented. Yet, an automated screening and simulation process is required to check the suitability of the numerous solutions. In [55] the previous model is refined to include multiple clutches operations.

In [20] an asymmetric (see Fig. 4) layout is studied. It is shown that the Willis' ratio can be obtained by imposing some functional constraints. However, no ordinary gears are involved, and the calculation of the efficiency rely on a quite laborious power flow analysis. A similar approach is also performed in [21] for a simple shunted e-CVT, and in [22] for another asymmetric e-CVT.

Ultimately, there exist several exhaustive models for the design of such transmissions, most of them relying on an explorative approach. Others seem more selective, but a slight increase in constructive complexity could make them unreasonably tedious.

Our aim is to provide instead a quick tool, which consents to assess and understand the influence of simple functional parameters on the overall performances. Accordingly, our design model primarily directs the engineer towards the most performant solutions, which have to be discriminated on the bases of clear constructive constraints.

The theory is the evolution of the works [29,30,32,37] and [57], from which it also takes most of its mathematical symbolism. Within this paper, we will perform the kinetostatic design of a power-split CVT comprising two (or one) planetary gear trains, up to six ordinary joints and one variator (of any kind).

The design process is modular, allowing the engineer to achieve awareness of the transmission operation for subsequent steps and to optimize the efficiency of the main devices without mutual interference and in accordance with a specific priority order, which is:

1. to enhance the overall efficiency and minimize the power class of the variator;
2. to let the planetary gear trains work near their synchronism;
3. to evaluate the available layouts and select the most appropriate one;
4. to arrange the ordinary gear sets.

Three functional parameters determine the power flow into the variator, allowing the engineer to estimate its power class regardless the actual arrangement of the transmission. The possible layouts are suggested by the method itself only in the second step, in terms acts to guarantee to the planetary gear trains the constructive feasibility and a synchronous condition internal to the range of operation. The internal power flows, contribute, in the final step, to the choice of the number, arrangement and speed ratios of the necessary ordinary joints in order to match together the absolute speeds.

2. Theory

Conceptually, whatever Power-Split Continuously Variable Transmission (PS-CVT) consists of a Continuously Variable Unit (CVU) and of a Power-Split Unit (PSU) (Fig. 1).

The CVU is a device with two mechanical outputs, whose speeds are controlled. It could be a simple mechanical variator (V-Belt variator, toroidal variator, etc.) with one degree of freedom (DOF), as well as a complex hydraulic or electric system (including motors/generators, energy storage systems etc.) with two DOF.

The PSU can be outlined as a *four-port differential* (Fig. 2), i.e. a mechanical driveline with four output shafts and two DOF [16]. Each port coincides with one of the *main* shafts, i.e. the input and output shafts of the CVU (namely "*i*" and "*o*")

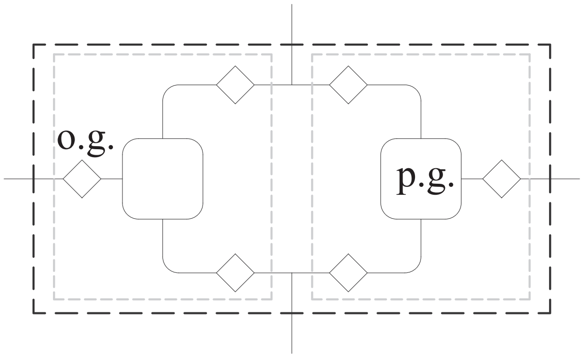

Fig. 2. The general PSU consists of two three-port differentials, each one made up of a planetary gearing (p.g.) and three ordinary gears (o.g.). There are two pairs of homologous joints, i.e. ordinary gears linked to the same main shaft.

and of the whole transmission (namely "*IN*" and "*OUT*"). Hereinafter, it consists of two planetary gear trains and up to six ordinary gears, (Section 4 describes the case with only one planetary gearing).

2.1. Kinematics

The angular speed of each PSU's shaft (internal or external) can be expressed as a linear function of two others, such as the overall input speed ω_{IN} and the overall output speed ω_{OUT}. We define the generic jth speed ratio as usual:

$$\tau_j = \frac{\omega_j}{\omega_{IN}} = a_j + b_j\ \tau \tag{1}$$

In which τ is the overall speed ratio. However, using normalized speeds, defined as follows, is more suitable:

$$\tau'_j = \frac{\tau_j}{\tau_{jm}} = a'_j + b'_j\ \tau' \tag{2}$$

Indeed, within this paper, the apostrophe always denotes a variable normalized with respect to its initial value, which is marked with the subscript $(\ \cdot\ )_m$ in order to point out its concurrency with the overall initial transmission ratio τ_m; similarly, we mark its end value with the subscript $(\ \cdot\ )_M$ in order to point out its concurrency with the overall final transmission ratio τ_M.

Accordingly the CVU's normalized transmission ratio τ'_v is:

$$\tau'_v = \frac{\tau'_o}{\tau'_i} = \frac{\tau_o}{\tau_i} \cdot \frac{\tau_{i_m}}{\tau_{o_m}} = \frac{\tau_v}{\tau_{v_m}} \tag{3}$$

In addition, we define the overall ratio spread as:

$$A = \frac{\tau_M}{\tau_m} \tag{4}$$

Similarly, for the CVU it is:

$$A_i = \frac{\tau_{i_M}}{\tau_{i_m}} \qquad A_o = \frac{\tau_{o_M}}{\tau_{o_m}} \qquad A_v = \frac{\tau_{v_M}}{\tau_{v_m}} = \frac{A_o}{A_i} \tag{5}$$

The overall ratio spread A and two out of the three ratio spreads regarding the CVU (Eq. (5)), are the key parameters in order to design a compound CVT.

Indeed, for the i shaft at $(\ \cdot\ )_m$ operation point (i.e. for $\tau' = 1$) Eq. (2) gives

$$1 = a'_i + b'_i$$

while at $(\ \cdot\ )_M$ operation point (i.e. for $\tau' = A$)

$$A_i = a'_i + b'_i\ A$$

from which we obtain a'_i and b'_i, and then the normalized speed ratio τ'_i of the i main shaft is

$$\tau'_i = \frac{A - A_i}{A - 1} + \frac{A_i - 1}{A - 1} \cdot \tau' \tag{6}$$

Analogously for τ'_o it is

$$\tau'_o = \frac{A - A_o}{A - 1} + \frac{A_o - 1}{A - 1} \cdot \tau' \tag{7}$$

Alternatively, these relationships can be expressed as functions of the CVU's normalized speed ratio τ_v':

$$\tau' = \frac{A\left(\tau_v' - 1\right) + A_i\left(A_v - \tau_v'\right)}{\left(\tau_v' - 1\right) + A_i\left(A_v - \tau_v'\right)} \tag{8}$$

$$\tau_i' = \frac{A_i\left(A_v - 1\right)}{\left(\tau_v' - 1\right) + A_i\left(A_v - \tau_v'\right)} \tag{9}$$

$$\tau_o' = \tau_i' \cdot \tau_v' \tag{10}$$

2.2. CVU power flow

The fraction of power flowing through the input shaft "*i*" of the CVU system (see Fig. 1), which effects its power class and has a major impact on the overall efficiency, is defined as:

$$p_i = \frac{P_i}{P_{IN}} = \frac{T_i \cdot \omega_i}{T_{IN} \cdot \omega_{IN}} \tag{11}$$

Sorge et al. [32] obtained the following result by applying the principle of virtual work:

$$p_i = \frac{d\tau'/\tau'}{d\tau_v'/\tau_v'} = \frac{\left[(A - A_o) + (A_o - 1)\,\tau'\right]\left[(A - A_i) + (A_i - 1)\,\tau'\right]}{(A_o - A_i)\,(A - 1)\,\tau'} \tag{12}$$

Comparing Eq. (12) with Eqs. (6) and (7), the normalized relative power p_i' shows the following expression:

$$p_i' = \frac{p_i}{p_{i_m}} = \frac{\tau_o'\,\tau_i'}{\tau'} = \left(\tau_i'\right)^2 \cdot \frac{\tau_v'}{\tau'} \tag{13}$$

Accordingly, it is evident that a change in the sign of τ', τ_i' or τ_o' always follows a change in the sign of p_i; therefore PS-CVTs of reverse-forward speed type (namely IVT - Infinitely Variable Transmission) involve the use of CVUs that support the reversal of the power flow (for instance, a common motorcycle variator would not work). The sign of τ_i' and/or τ_o' changes if A_i and/or A_o are negative, which is not possible with a simple mechanical variator, but it is easily achievable with complex hydraulic or electric CVUs.

Moreover, if $\tau' \to 0$ (i.e. $\omega_{OUT} \to 0$) the ideal CVU power p_i diverges because of the no-losses hypothesis; however, if only CVU power losses are taken into account, the real CVU power $\overline{p_i}$ can be easily assessed in the whole operative range, by [57]:

$$\overline{p_i} = \frac{\overline{P}_i}{P_{IN}} = \left(\frac{A - A_o}{A - 1} \cdot \frac{1 - \eta_v}{\tau_o'} + \frac{1}{p_i}\right)^{-1} \tag{14}$$

in which p_i and τ_o' are known from Eq. (12) and (7). Accordingly, the overall efficiency η can be assessed by:

$$\eta = -\frac{\bar{P}_{OUT}}{P_{IN}} = -\bar{p}_{OUT} = 1 - \overline{p_i}\,\left(1 - \eta_v\right) \tag{15}$$

The legitimacy of Eqs. (14) and (15) is due to the high efficiency of gear/wrapping pairs and planetary gear sets (when working near their synchronous conditions).

Eqs. (14) and (15) have been obtained and experimentally validated in [57] for a passive CVU; nevertheless, they remain valid even though the CVU is an active device or has an energy storage system (as in a hybrid electric vehicle). In these cases, η_v represents the CVU's apparent efficiency, i.e. the negative ratio between the power flowing through the shaft "*o*" and the power flowing through the shaft "*i*" of the CVU, and the same applies to η; under these circumstances, both η_v and η could assume negative, or greater than one, values.

The overall speed ratio range A can be supposed a known design requirement at first, and therefore it is evident by Eq. (14) that A_o and A_i are the only parameters that can substantially modulate the power flowing through the CVU. As a result, in order to optimize the overall efficiency and the size of the CVU itself, the selection of the constructive layout is secondary to the choice of the optimal values for these ratio spreads, which, conversely, could be compromised. Generally speaking, the values of A_o and A_i should be that ones that ensure the best possible fitting between required and available CVU power in the whole operative range, while the values of τ_{o_m} and τ_{i_m} should be that ones that let the CVU operate in a proper speed range (see Section 3).

Anyway, as soon as the three ratio spreads A, A_i, A_o have been set, the CVU boundary conditions are defined, and the second critical components to be designed are the planetary gear trains, while the choice of the layout is still delayed.

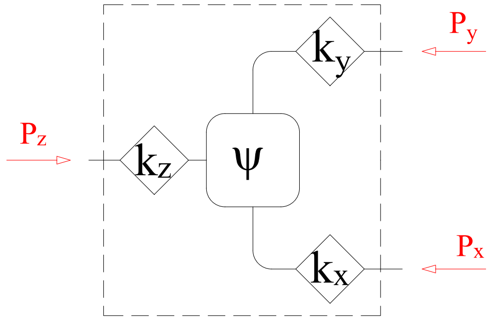

Fig. 3. Basic layout of a three-port differential, which is made up of one planetary gear train and three fixed ratio joints. Any power is assumed positive if entering.

2.3. Planetary gear sets and layouts

We define *three-port differential* a simple two DOF device, in which fixed-ratio joints (e.g. ordinary gears or any kinematically equivalent mechanisms) connect each *port* to one shaft of a planetary gear train (Fig. 3); within this paper, the *PSU* consists in a couple of *three-port differentials* sharing two distinct *ports* (Fig. 2).

Since planetary gear trains work with high efficiency near their synchronism, we suppose that such condition must be reached for a speed ratio τ_* internal to the working range. This efficiency-oriented assumption makes Eqs. (14) and (15) legitimate and the three ratio spreads A, A_i and A_o sufficient in order to guarantee the constructive feasibility of the planetary gear trains themselves, and to identify the most appropriate layout for the transmission.

Indeed, if k_j is the fixed ratio between the *jth* speed, which is external to the three-port differential (Fig. 3), and the matching internal shaft's speed, then imposing the synchronous conditions to the planetary gearing leads to the following relationships:

$$\left.\frac{\omega_x}{k_x}\right|_{\tau_*} = \left.\frac{\omega_y}{k_y}\right|_{\tau_*} = \left.\frac{\omega_z}{k_z}\right|_{\tau_*} \tag{16}$$

valid for any position of carrier, sun and ring gear.

A simple planetary gear train is conventionally defined by its *constructive parameter*, also known as the Willis' ratio or the characteristic ratio, i.e. the ratio between the speed of the ring over the speed of the sun when the carrier is motionless, and so we have to write:

$$\psi^{\mathrm{carr.}}_{\mathrm{ring/sun}} = \frac{\omega_{ring} - \omega_{carr}}{\omega_{sun} - \omega_{carr}} = \left.\frac{\omega_{ring}}{\omega_{sun}}\right|_{\omega_{carr}=0}$$

However, since we are working with the normalized exit speeds of the tree port differential, then we can write it as:

$$\psi^{\mathrm{carr.}}_{\mathrm{ring/sun}} = \left.\left(\frac{\omega_r}{\mathrm{k_r}} \cdot \frac{\mathrm{k_s}}{\omega_s}\right)\right|_{\omega_c/\mathrm{k_c}=0} = \frac{\mathrm{k_s}}{\mathrm{k_r}} \cdot \left.\frac{\omega_r}{\omega_s}\right|_{\omega_c=0} = \left.\frac{\tau_s'}{\tau_r'}\right|_{\tau_*'} \cdot \left.\frac{\tau_r'}{\tau_s'}\right|_{\tau_c'=0} \tag{17}$$

in which k_s is the fixed ratio between the speed ω_s of the shaft of three port differential linked to the sun gear, and the speed of the sun gear itself (let's name it ω_{sun}), i.e. $\omega_s/\omega_{sun} = k_s$. Similarly, $\omega_r/\omega_{ring} = k_r$. As stated before, the arrangement of the shafts of the planetary gear trains and their synchronous points are not definite at this stage. Accordingly, we use x,y,z (Fig. 3) to represent all the different possible combinations and to define, in analogy to Eq. (17), a generic *characteristic function* as follows:

$$\phi^z_{y/x}(\tau') = \left.\frac{\tau_x'}{\tau_y'}\right|_{\tau'} \cdot \left.\frac{\tau_y'}{\tau_x'}\right|_{\tau_z'=0} \tag{18}$$

which is valid in the whole operation range.

Whatever it is the layout of the PSU (Fig. 2), each three-port differential (Fig. 3) is linked to three out of the four main shafts; therefore, we can identify four possible typologies of three-port differentials (hereinafter named groups) depending on the involved main shafts (Table 1).

For each group we get six *characteristic functions* by permuting the three shaft indexes in Eq. (18). Formerly, each value of a *characteristic function* is a possible Willis' ratio providing synchronism. Since the four main normalized speeds are known, all the characteristic functions are defined, and each group can suggest six Willis' ratios (but only three actually different, due to a changed position of the planet-carrier) for a given synchronous working point τ_*'. For instance, the function $\phi^i_{OUT/IN}$

Table 1
Typologies of three-port differentials and main shafts involved

Group	Shafts
C_{IN}	$i,\ o,\ IN$
C_{OUT}	$i,\ o,\ OUT$
D_i	IN, OUT, i
D_o	$IN, OUT,\ o$

Table 2
Characteristic functions: $\phi^z_{y/x}(\tau')$ represents possible Willis' ratios providing synchronism and the power ratio $-P_x/P_y$.

Group	Characteristic functions					
$\boldsymbol{D_i}$	$\phi^i_{OUT/IN} = \alpha_i/\tau'$	$\phi^i_{IN/OUT}$	$\phi^{IN}_{OUT/i}$	$\phi^{IN}_{i/OUT}$	$\phi^{OUT}_{IN/i}$	$\phi^{OUT}_{i/IN}$
$\boldsymbol{D_o}$	$\phi^o_{OUT/IN} = \alpha_o/\tau'$	$\phi^o_{IN/OUT}$	$\phi^{IN}_{OUT/o}$	$\phi^{IN}_{o/OUT}$	$\phi^{OUT}_{IN/o}$	$\phi^{OUT}_{o/IN}$
$\boldsymbol{C_{IN}}$	$\phi^{IN}_{o/i} = \frac{1-\alpha_i/\tau'}{1-\alpha_o/\tau'}$	$\phi^{IN}_{i/o}$	$\phi^i_{o/IN}$	$\phi^i_{IN/o}$	$\phi^o_{i/IN}$	$\phi^o_{IN/i}$
$\boldsymbol{C_{OUT}}$	$\phi^{OUT}_{o/i} = \frac{1-(\alpha_i/\tau')^{-1}}{1-(\alpha_o/\tau')^{-1}}$	$\phi^{OUT}_{i/o}$	$\phi^i_{o/OUT}$	$\phi^i_{OUT/o}$	$\phi^o_{i/OUT}$	$\phi^o_{OUT/i}$
	φ	$\frac{1}{\varphi}$	$1-\varphi$	$\frac{1}{1-\varphi}$	$1-\frac{1}{\varphi}$	$\frac{-\varphi}{1-\varphi}$

of the D_i group is:

$$\phi^i_{OUT/IN}(\tau') = \left.\frac{1}{\tau'}\right|_{\tau'} \cdot \left.\frac{\tau'}{1}\right|_{\tau_i'=0} = -\frac{A-A_i}{(A_i-1)\ \tau'} \tag{19}$$

and if it is $\psi = \phi^i_{OUT/IN}(\tau'_*)$ the carrier is linked to the "*i*" main shaft of the four port differential, while the ring gear is linked to "*OUT*" and the sun gear is linked to "*IN*".

The other 23 characteristic functions could be calculated similarly. However, it is actually necessary to use Eq. (18) once for each group, while the other five functions (which are in the same group) can be easily calculated by the use of the well-known relationships reported in the last line of Table 2, which details all the 24 characteristic functions in a brief way. For this purpose, we define two simple implicit functional parameters as follows:

$$\alpha_i = \frac{A_i - A}{A_i - 1} \qquad\qquad \alpha_o = \frac{A_o - A}{A_o - 1} \tag{20}$$

Formerly, they represent the normalized overall transmission ratios for which respectively τ_i or τ_o is null (see Eqs. (6) and (7)). However, we prefer this notation to point out that they may not have physical meaning, i.e. represent a possible working point.

Even the equations from Eqs. (6) to (15) are functions of these two parameters only (as stated before, the PSU has two DOF). However, for these last we prefer to keep the current explicit form, because they always maintain a direct physical meaning, and therefore are more intelligible.

Therefore, in order to explore the feasible layouts (for given A, A_i, A_o), and then select the most appropriate one, it is convenient to perform the following steps:

- to build a *design-chart* by plotting all the characteristic functions in the whole range of τ', from one to A (see Fig. 5);
- to discard those curves that do not possess values within the desired range of Willis' ratios;
- to select from two characteristic curves, belonging to different groups, two points (i.e. the Willis' ratios and the related synchronous working points).

Definitively, to choose the Willis' ratio from a specific curve determines the position of the carrier, sun and ring of each planetary gear train, while the groups to which these functions belong determine the final layout. Section 3.3 reports a brief example on how to use a design-chart properly.

Generally speaking, there are $\binom{4}{2} = 6$ possible schemes, two of which are symmetric (Fig. 4).

With reference to Table 1, we choose the letters *C* and *D* because they permit to identify the *fundamental layouts* of Fig. 4. Two three-ports differentials both belonging to a *D* group will combine in a "Direct bridge" symmetric layout in which the CVU directly connects the two three-port differentials, while two three-ports differentials both belonging to a *C* group will combine in a "Cross bridge" symmetric layout. Conversely, one *C* and one *D* group always combine in an Asymmetric Bridge layout.

In addition, the subscript indicates the different shaft between groups with the same capital letter: so C_{IN} and C_{OUT} both include *i* and *o*, but only C_{IN} include *IN*, while C_{OUT} include *OUT; D_i* and D_O both include *IN* and *OUT*, but only D_i includes *i*, while D_O includes *o*.

It is enough in order to identify a specific *functional scheme* of Fig. 4.

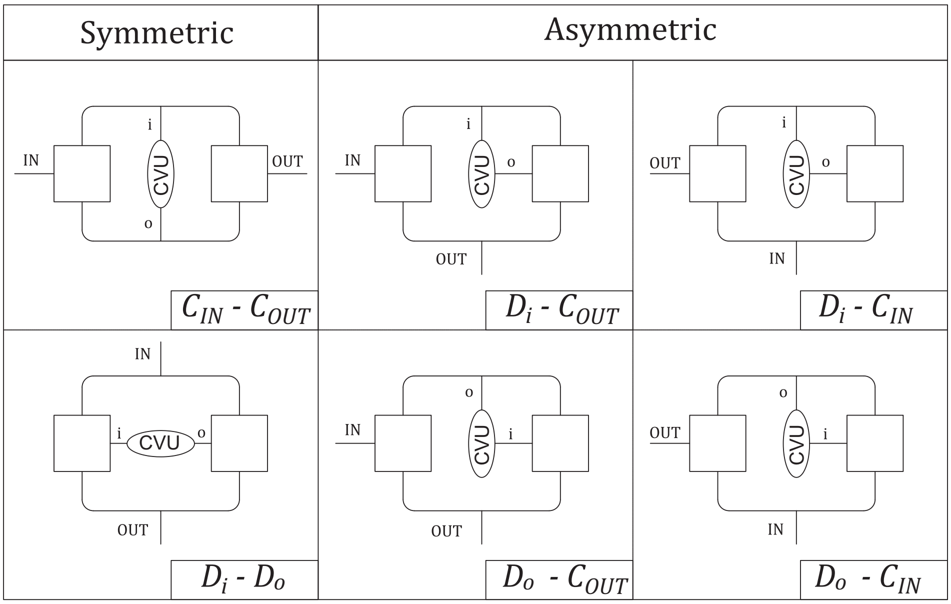

Fig. 4. Functional schemes for compound power-split transmissions. The fundamental layouts are: Cross Bridge, Direct Bridge and Asymmetric Bridge.

2.4. Internal power flows

Assessing the internal power flows is significant because it can lead the designer to smart and efficiency-oriented choices, such as identifying the optimal arrangement for the ordinary gears. Polder [27] originally introduced the kinematical concept of *responsivity*, which is one of the best mathematical tools for estimating the power flowing through the variable speed unit. However, within this article, we suggest a different technique for the assessment of the remaining power flows.

The power ratio between two out of the three shafts of a three-port differential is:

$$\frac{P_x}{P_y} = \frac{T_x}{T_y} \cdot \left.\frac{\omega_x}{\omega_y}\right|_{\tau} \tag{21}$$

Since the torque ratio is constant regardless of the output speeds, neglecting power losses into the subsystem leads to the following mathematical relationship:

$$\frac{T_x}{T_y} \cdot \left.\frac{\omega_x}{\omega_y}\right|_{\omega_z=0} = -1 \tag{22}$$

As a result, the previous expression (21) becomes:

$$\frac{P_x}{P_y} = -\left.\frac{\omega_x}{\omega_y}\right|_{\tau} \cdot \left.\frac{\omega_y}{\omega_x}\right|_{\omega_z=0} = -\left.\frac{\tau'_x}{\tau'_y}\right|_{\tau'} \cdot \left.\frac{\tau'_y}{\tau'_x}\right|_{\tau'_z=0} \tag{23}$$

Accordingly, the above-defined *characteristic functions* also represent the opposite of the ratio between the powers transmitted through two shafts of a three-port differential:

$$\phi^z_{y/x}(\tau') = -\frac{P_x}{P_y}(\tau') = \left.\frac{\tau'_x}{\tau'_y}\right|_{\tau'} \cdot \left.\frac{\tau'_y}{\tau'_x}\right|_{\tau'_z=0} \tag{24}$$

Since the four main powers are known (Eqs. (14) and (15)) and a three-port differential has always a port exclusively linked to one of the main shafts (see Fig. 4), then the powers on the other two ports can be calculated directly by some of the functions in Table 2, without the need to solve large systems of equations.

The duality of the *characteristic functions* provides, by means of their graphical representation (*design-chart*), an excellent overview of the transmission at the earliest stages of study, when only the essential functional design parameters are defined (A, A_i, A_o), leading the designer to conscious and efficiency-oriented solutions.

2.5. Ordinary gears

Imposing a synchronous condition to the planetary gear train of a *three-port differential* (see Eq. (16)) implies the following two (mutually independent) constraints for the speed ratios of its ordinary gear trains (or any equivalent fixed-ratio

Table 3
Design constraints for the ordinary gears. The implicit functional variables α_i and α_o are defined by Eq. (20).

Groups	Constraint	Expression
C_{IN}, D_i	$\frac{k_i}{k_{IN}}$	$\tau_{i_m} \cdot \frac{\tau'_* - \alpha_i}{1-\alpha_i}$
C_{IN}, D_o	$\frac{k_o}{k_{IN}}$	$\tau_{o_m} \cdot \frac{\tau'_* - \alpha_o}{1-\alpha_o}$
C_{OUT}, D_i	$\frac{k_i}{k_{OUT}}$	$\frac{\tau_{i_m}}{\tau_m\ \tau'_*} \cdot \frac{\tau'_* - \alpha_i}{1-\alpha_i}$
C_{OUT}, D_o	$\frac{k_o}{k_{OUT}}$	$\frac{\tau_{o_m}}{\tau_m\ \tau'_*} \cdot \frac{\tau'_* - \alpha_o}{1-\alpha_o}$

joint):

$$\frac{k_x}{k_y} = \frac{\tau_x}{\tau_y}\bigg|_{\tau'_*} \tag{25}$$

$$\frac{k_x}{k_z} = \frac{\tau_x}{\tau_z}\bigg|_{\tau'_*} \tag{26}$$

For given $A,\ A_i, A_o, \tau'_*$, in each *three-port differential* of Table 1, these constraints are proportional to some of the three initial speed ratios, i.e. τ_m, τ_{i_m} and τ_{om}. E.g., for the ordinary gears of a C_{IN} *three-port differential,* one constraint is:

$$\frac{k_i}{k_{IN}} = \frac{\tau_i}{1}\bigg|_{\tau'_*} = \tau_{i_m} \cdot \frac{(A_i - 1)\ \tau'_* + A - A_i}{A-1} \tag{27}$$

Table 3 lists the constraints for the fixed-ratio joints of each group: different groups share the same expression of the constraint between *homologous joints*, i.e. joints linked to the common main shafts, but belonging to different branches of the transmission (see Fig. 2).

Completely tuning the kinematic of a compound transmission, i.e. imposing the initial ratios $\tau_m,\ \tau_{o_m}, \tau_{i_m}$, implies four different design constraints, and then requires, generally speaking, at least four ordinary joints. This result can be obtained by:

- eliminating one ordinary joint in each *three-port differential*;
- eliminating only one ordinary joint and imposing to a pair of *homologous joints* the same value, so that they can be merged in just one.

In the latter case, if the two planetary gear trains share the same synchronous point τ'_*, even the other two *homologous joints* result identical to each other (see Table 3); accordingly, they can be merged too, and only three ordinary joints could be sufficient in this instance.

In general, to eliminate one or more ordinary gears is possible if the remaining fixed ratios are feasible; anyway, the actual number and arrangement of these gears should be driven by constructive issues and power flows analysis (see Section 3.3).

3. Design guidelines

3.1. About the ratio spreads

As stated in Section 2.2, the values of A_o and A_i should be the ones that guarantee the best fit between deliverable and delivered power, which implies a good sizing of the CVU. Since this choice can be strongly influenced by the CVU's apparent efficiency maps (which might model both the real efficiency and the presence of an energy storage system with its control strategy), it is clear that a rigorous choice of these parameters is possible only for a specific study case.

However, since this work is aimed to generality, hereinafter we provide some simple general guidelines assuming $\eta_v = 1$ (i.e. no power losses and no energy storage or active systems), as the obtained results could be used anyway as guess values for future refinements.

The values of A_i and A_o are both free for Electric or Hydraulic CVU, while for a simple mechanical CVU they are positive and their ratio A_v is restricted by constructive issues. Therefore, we will examine these cases separately.

3.1.1. Electric or hydraulic CVU

To select negative ratio spreads for A_i and/or A_o determines up to two overall speed ratios ($\tau'_{\#,i}$ and/or $\tau'_{\#,o}$) for which p_i is null (the so-called "mechanical points"), as well as τ'_i or τ'_o (see Eqs. (6) and (7)). In these circumstances, it must be:

$$A_i = -\left(\frac{A - \tau'_{\#,i}}{\tau'_{\#,i} - 1}\right) \tag{28}$$

and/or:

$$A_o = -\left(\frac{A - \tau'_{\#,o}}{\tau'_{\#,o} - 1}\right) \tag{29}$$

Anyway, if there are not privileged speed ratios for the mechanical points ($\tau'_{\#,i}$, $\tau'_{\#,o}$) and $A > 0$, it could be useful to minimize p_i in the whole operation range (if $A < 0$, near the neutral gear, p_i tends to high values regardless of A, A_i, A_o). After some math, it is possible to prove that the following condition:

$$A_i = -\sqrt{A/A_v} \text{ and } A_o = -\sqrt{A\, A_v}$$
$$\text{with } A_v = (a \pm \sqrt{a^2 - 1})^2 \text{ and } a = \frac{(1+\sqrt{A})^2}{2\sqrt{A}} + 1$$

makes $p_{i_m} = -p_i(\sqrt{A}) = p_{i_M}$, and then max $|p_i|$, valued in the whole operation range, the least possible. This implies the presence of two mechanical points, which can be identified by inverting Eqs. (28) and (29), i.e. through Eq. (20).

3.1.2. Simple mechanical CVU

If both A_o and A_i must be positive and their ratio A_v is restricted by constructive issues (e.g. when using a V-belt variator), it is no longer possible to achieve a mechanical point. Since at the ends of the working range, p_i is:

$$p_{i_m} = \frac{1}{A_i} \cdot \frac{A-1}{A_v - 1} \tag{30}$$

$$p_{i_M} = \frac{A_i\, A_v}{A} \cdot \frac{A-1}{A_v - 1} \tag{31}$$

The following condition makes equal their absolute values:

$$A_i = \pm\sqrt{\left|\frac{A}{A_v}\right|} \tag{32}$$

and since the product of p_{i_m} and p_{i_M} does not depends on A_i, the previous relationship (32) is also that one that minimizes p_i at its ends. As an alternative, the value of A_i that minimize the absolute value of $|p_i|$ at the specific working point $\tau'_\#$ is:

$$A_i = \sqrt{\frac{1}{A_v}} \cdot \left(\frac{A - \tau'_\#}{\tau'_\# - 1}\right) \tag{33}$$

An interesting result is that the previous condition is reached for the same CVU's normalized ratio, which is function of A_v only: indeed, Eqs. (33) and (8) lead to a relationship that is always fulfilled when:

$$\tau'_{v_\#} = \sqrt{A_v} \tag{34}$$

The previous criteria (Eqs. (33) and (34)) are well suited for common mechanical variators. Anyway, when using such variators, the choice of extreme values for A_i (i.e. much smaller or bigger than one) should be avoided, as it can negatively impact the sizing of the CVU by reducing too much the available minimum speed of its input shaft, or require too sudden changes of τ_v.

Moreover, by using the previous criterion (Eq. (32)), we favor indirectly the following overall speed ratio (see Eq. (34)):

$$\tau'_\# = \frac{A + \sqrt{|A|}}{1 + \sqrt{|A|}} \tag{35}$$

which is close to the stall speed for $A \to -1$ or simply equal to $\sqrt{A}$ if A is positive. If A is positive, $p_i(\sqrt{A})$ represents a local minimum or maximum, but, in any case, with the least possible value, which implies that $\max|p_i(\tau')|$, valued in the whole operation range, is the least possible for the available CVU's ratio spread A_v. If $A \to -1$, it should improve p_i near the neutral gear, where it tends to high values.

3.2. About the initial speed ratios

At first stage of the design process, τ_m is a known parameter (as well as the overall ratio spread A). On the contrary the values of τ_{o_m} and τ_{i_m} should be evaluated in order to make available enough CVU's power capacity in the whole operation range, while working in an efficient speed range. As a general rule in order to limit torques, their absolute values should be as great as possible, and if Ω_j represent the maximum magnitude for the $j-th-$speed of the CVU, it should be

$$|\tau_{j_m}| \leq \Omega_j / \max\left|\omega_{IN}\ \tau'_j\right|. \tag{36}$$

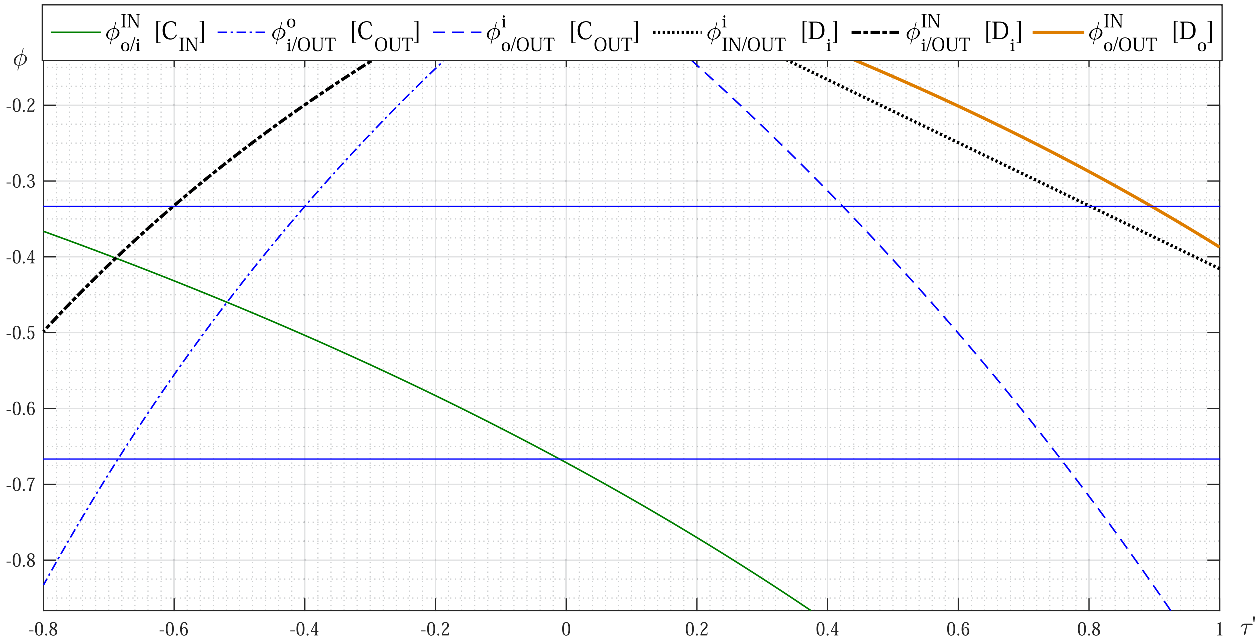

Fig. 5. Design-chart for $A = -0.8$, $A_v = 3.6$, $A_i = 0.47$ and Willis' ratios within the range [−1/3,−2/3]. It represents the characteristic functions ϕ. Their superscripts and subscripts define the carrier, ring and sun position.

When using a mechanical variator, the CVU's initial ratio $\tau_{v_m} = \tau_{o_m}/\tau_{i_m}$ is usually a known parameter; the choice of the speed ratio $|\tau_{im}|$ affects both the input torque and the absolute speeds of the CVU, while its sign affect the constraints of the fixed-ratio joints (Table 3).

If $A \le 0$, the applied fraction of ideal torque θ_i would tend to $\pm\infty$ when approaching the overall stall speed. In this case, it is mandatory to assess accurately the related CVU's power ratio η_v (see Section 2.2), calculating the real fraction of torque by:

$$\overline{\theta_i}\Big|_{\tau'=0} = \frac{\overline{p_i}}{\tau_i}\bigg|_{\tau'=0} = \frac{1}{\tau_{i_m}} \cdot \frac{A-1}{A-A_i} \cdot \frac{1}{1-\eta_v} \tag{37}$$

3.3. Application

In order to highlight the effectiveness of the above design method, a brief example of application is performed.

An IVT with an overall transmission ratio τ ranging from −0.5 to 0.4 has to be designed, the available CVU is mechanical and its ratio τ_v varies from 0.53 to 1.9, while the engine runs at a fixed speed. As a result, the known functional parameters are $\tau_m = -0.5$, $A = -0.8$, $\tau_{v_m} = 0.53$, $A_v = 3.6$.

In order to smooth out performances, according to Eq. (32), we impose that $A_i = 0.47$. From Eq. (35) it follows that this choice enhances also the performances near the neutral gear condition, as it optimizes $\tau_\# = -0.025$.

If the maximum tolerable value for τ_{i_m} is ∓ 2.5, with an average CVU's low-load efficiency of $\eta_v = 0.75$ for the forward overall speed ratios (since p_i is positive, the power enters in i and exits from o), then the maximum CVU input torque ratio (see Eq. (37)) is about $|\overline{\theta_i}|_{\tau=0^+} = 2.3$.

If we assume that the Willis' ratios must be within the range $-2/3 \div -1/3$ (positive simple planetary gear trains can offer a wider $\psi^{\mathrm{carr}}_{\mathrm{sun2/sun1}}$ range, with values much closer to one, but usually show a lower efficiency [56]), for $A = -0.8$, $A_v = 3.6$ and $A_i = 0.47$, we obtain the design-chart in Fig. 5, where 18 out of the 24 characteristic functions of Table 2 do not meet the previous requirement.

In general, the Willis' ratios and the synchronous working points of the two planetary gear trains are different. However, if the selected points on the characteristic curves can share the same ordinate, the planetary gear trains have identical Willis' ratios, while if they can share the same abscissa, then the planetary gear trains have the same synchronous working point, and it could be possible to reduce the number of ordinary gears. At times, both the previous conditions can be fulfilled, with the resulting advantages.

In our example, the latter circumstance occurs twice. The characteristic curves $\phi^{IN}_{\mathrm{o/i}}$ and $\phi^{o}_{\mathrm{i/OUT}}$ intersect in $\tau'_* = -0.52$ and $\psi = -0.46$, leading to a $C_{IN} - C_{OUT}$ cross-bridge layout with the carriers linked to the "IN" and "o" shafts respectively, while $\phi^{IN}_{\mathrm{i/OUT}}$ and $\phi^{IN}_{\mathrm{o/i}}$ intersect in $\tau'_* = -0.69$ and $\psi = -0.40$, leading to a $D_i - C_{IN}$ asymmetric-bridge layout with both

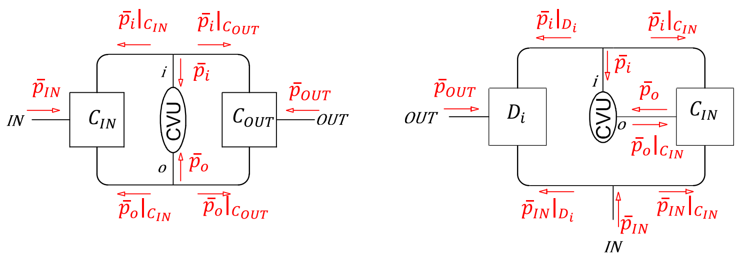

Fig. 6. Selected functional schemes for the transmission of the example of Section 3.3. According to Fig. 1 and Fig. 3 all the represented power flows are assumed to be positive.

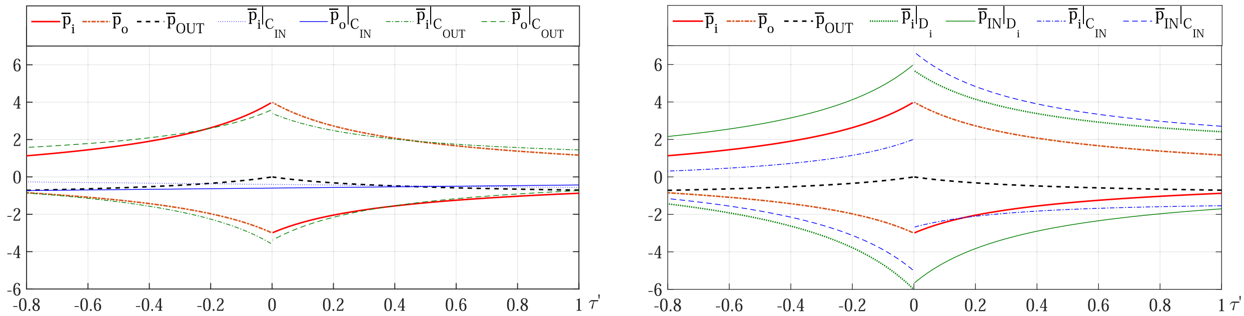

Fig. 7. Internal power flows of the $C_{IN}-C_{OUT}$ (left) and D_i-C_{IN} (right) layouts for the transmission of the example of Section 4 ($A=-0.8, A_v=3.6, A_i=0.47, \eta_v=0.75$).

Table 4
Design constraints for the ordinary gears of the example of Section 3.3 ($A=-0.8$, $\tau_m=-0.5$ $A_v=3.6$, $\tau_{v_m}=0.53$, $A_i=0.47$, $\tau_{i_m}=-2.5$ and $\tau'_*|_{C_{IN}}=\tau'_*|_{C_{OUT}}=-0.52$).

Constraint	Value
$(k_i/k_{IN})\|_{C_{IN}}$	-1.38
$(k_i/k_{OUT})\|_{C_{OUT}}$	-5.34
$(k_o/k_i)\|_{C_{IN}}$	1.52
$(k_o/k_i)\|_{C_{OUT}}$	1.52

the carriers linked to the "IN" port. Fig. 6 shows both these functional schemes. According to the theory, any power is assumed positive if entering.

In order to discern between these two eligible solutions, it is useful assessing the internal power flows.

For the D_i-C_{IN} layout, the ports with an exclusive link to a main shaft are "OUT" for D_i and "o" for C_{IN}. Accordingly, the internal power flows are:

$$\bar{p}_{OUT}|_{D_i}=\bar{p}_{OUT}=-\eta \qquad \bar{p}_o|_{C_{IN}}=-\bar{p}_o=\eta_v\ \bar{p}_i$$

$$\bar{p}_i|_{D_i}=-\bar{p}_{OUT}|_{D_i}\cdot\phi^{IN}_{OUT/i}=\ \eta\ \phi^{IN}_{OUT/i} \qquad \bar{p}_i|_{C_{IN}}=-\ \bar{p}_o|_{C_{IN}}\cdot\phi^{IN}_{o/i}=-\eta_v\ \bar{p}_i\ \phi^{IN}_{o/i}$$

$$\bar{p}_{IN}|_{D_i}=-\bar{p}_{OUT}|_{D_i}\cdot\phi^{i}_{OUT/IN}=\ \eta\ \phi^{i}_{OUT/IN} \qquad \bar{p}_{IN}|_{C_{IN}}=-\ \bar{p}_o|_{C_{IN}}\cdot\phi^{i}_{o/IN}=-\eta_v\ \bar{p}_i\ \phi^{i}_{o/IN}$$

For the $C_{IN}\ -C_{OUT}$ layout, the ports with an exclusive link to a main shaft are "IN" for C_i and "OUT" for C_{OUT}, but the procedure is very similar and is omitted here.

Fig. 7 shows the diagrams of the power flows for the above solutions. The $C_{IN}-C_{OUT}$ layout seems preferable, as all the internal power flows are lower on average.

Therefore, we calculate the necessary fixed-ratio joints only for this layout. Their kinematic constraints, calculated through Table 3, are shown in Table 4.

Since there is a common synchronous point, the *homologous joints* share the same constraint. Therefore, we can merge them in just one, and only three ordinary joints are strictly needed. Accordingly, we can impose that it is $k_i|_{C_{IN}}=k_i|_{OUT}=k_i$ and $k_o|_{C_{IN}}=k_o|_{C_{OUT}}=k_o$, obtaining the results in Table 5. The first solution (first row in Table 5) requires fixed ratios that are easily realizable with a single-stage gearing. However, the second one appears to be a little more efficient, as $|\bar{p}_i|$ is mostly bigger than one (see Fig. 7); accordingly, if working near the neutral gear were particularly important, a two-stage

Table 5
Possible values for the fixed ratios of the example of Section 3.3 ($A = -0.8$, $\tau_m = -0.5A_v = 3.6$, $\tau_{v_m} = 0.53$, $A_i = 0.47$, $\tau_{i_m} = -2.5$ and $\tau'_*|_{C_{IN}} = \tau'_*|_{C_{OUT}} = -0.52$. For $\tau_{i_m} = +2.5$ all the previous values would be positive).

k_{IN}	k_i	k_o	k_{OUT}
1	–1.38	–2.10	0.258
–0.725	1	1.52	–0.187
–0.476	0.658	1	–0.123
3.87	–5.34	–8.11	1

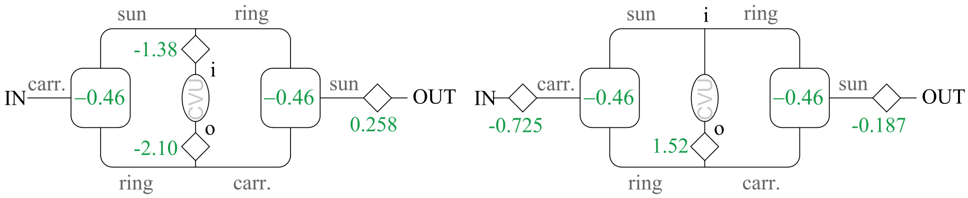

Fig. 8. Eligible final constructive schemes of the transmission designed in Section 3.3. For $\tau_{i_m} = +2.5$ all the fixed ratios would be positive.

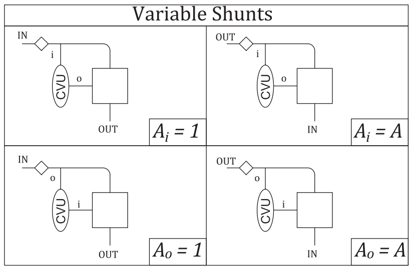

Fig. 9. Variable shunt transmissions: Output-splits (on the left) and input-split (on the right).

k_{OUT} could be considered still viable. The possible final constructive schematics of the designed transmission are shown in Fig. 8.

Small variations between target and realizable constructive ratios will bring to slightly different functional parameters as shown in Section 4.1.

4. Variable shunt transmissions

The presented model, developed for a compound transmission, results still valid even though only one planetary gear train is employed, i.e. for the so-called variable shunt transmissions. In these cases, two out of the four main shafts have proportional speeds and, therefore, the same ratio spread, as shown in Fig. 9.

Accordingly, if during the design process one of the conditions in Fig. 9 happens to be almost fulfilled for A_i or A_o, then the compound layout can be avoided at all.

In these cases, the characteristic functions of different groups appear overlapped in the design chart, and it is sufficient to:

- choose one Willis' ratio (or the related synchronous working point);
- constraint the ordinary gears of the remaining three-port differential by two of the expressions of Table 3;
- introduce an additional ordinary joint between the two connected main shafts in order to satisfy the relationship between their respective initial speed ratios.

4.1. Application

An IVT with an overall transmission ratio τ ranging from -0.27 to 0.95 has to be designed by means of an electric CVU; the electric machine linked to "*i*" has $max|\tau_i| = 6.5$ and has to give a mechanical point for $\tau_{\#,i} = 0.34$, vice versa the electric machine linked to "*o*" has $max|\tau_o| = 3.8$ has to give a mechanical point for $\tau_{\#,o} = 0$. From Eqs. (28) and (29), we get: $A_o = A = -3.5$ and $A_i = -1.0$, so τ_i varies from 6.5 to -6.5 and τ_o from -1.08 to 3.80.

In the design chart (Fig. 10), the characteristic functions of the D_i and C_{IN} group result overlapped. Indeed, the main speeds τ and τ_o are proportional (τ' and τ'_o coincide), and therefore one planetary gearing can be eliminated at all. A variable shunt of input-split type is obtained (see Fig . 9).

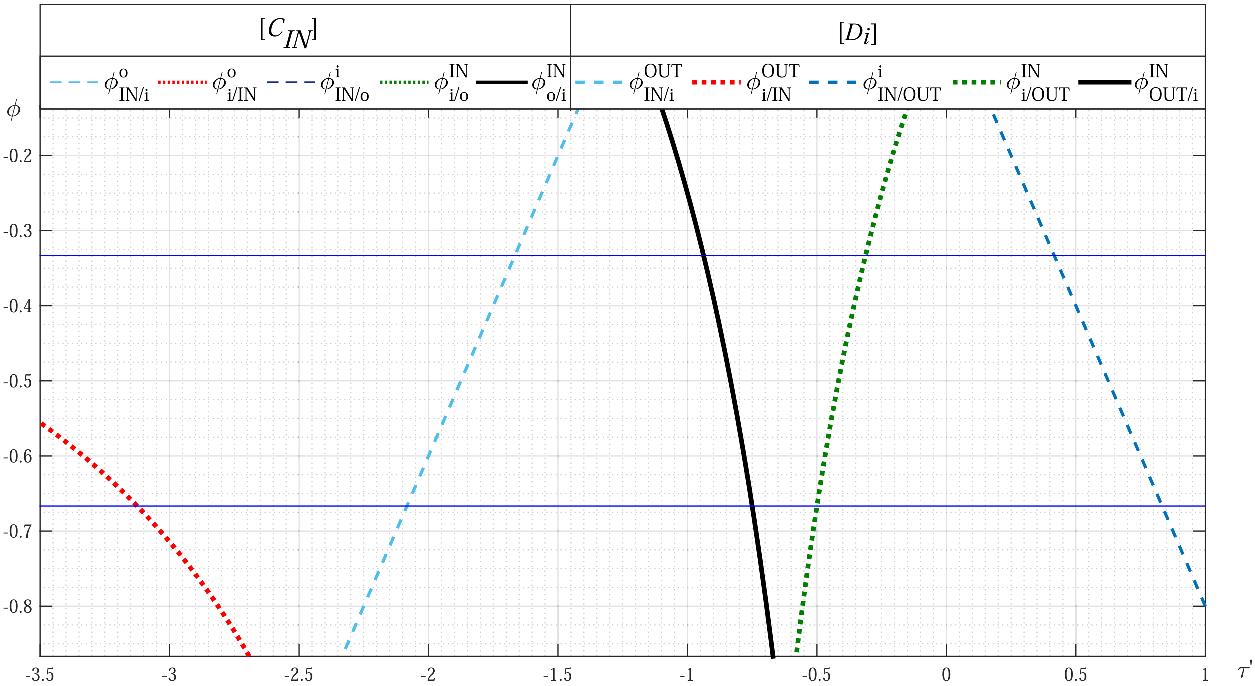

Fig. 10. Design chart for the PS-CVT with $A = A_o = -3.5, A_i = -1$ and Willis' ratios within the range $[-1/3,-2/3]$. It represents the characteristic functions ϕ. Their superscripts and subscripts define the carrier, ring and sun position.

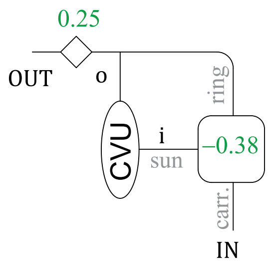

Fig. 11. Final constructive scheme of the transmission designed in Section 4.1.

The choice of τ'_* do not affect the internal power flows. In particular, if we chose that the synchronous condition is reached for $\tau'_* = 0.91$, we obtain that the Willis' ratio is $\psi^{IN}_{o/i} = \phi^{IN}_{o/i}(-0.91) = -0.38$ (see Fig. 10). From the equations of Table 3 we notice that this conditions lead also to $\frac{k_i}{k_{IN}} \approx \frac{k_o}{k_{IN}} \approx 1$, which implies that also the ordinary gearing k_{IN}, k_i and k_o can be eliminated. The lasting ordinary gearing is simply $k_{OUT} = \frac{\tau_m}{\tau_{om}} = \frac{-0.27}{-1.1} = 0.25$ (see Fig. 11).

This result is indeed very similar to the constructive scheme of the first generation of Toyota Prius' PSD, in which $\psi^{IN}_{o/i} = -\frac{30}{78} = -0.385$, $k_i = k_o = k_{IN} = 1$ and $k_{OUT} = \frac{36}{35} \cdot \frac{30}{44} \cdot \frac{26}{75} = 0.243$. In this case, the overall transmission ratio τ ranges from -0.27 to 0.94, with $\tau_i = 6.5 \div -6.5$ and $\tau_o = -1.1 \div 3.9$, $\tau_{\#,o} = 0$, $\tau_{\#,i} = 0.34$ and $\tau'_* = 0.90$.

5. Results and discussion

The normalized speed ratios τ', τ'_i and τ'_o, which vary from one to their respective ratio spreads A, A_i, and A_o, greatly affect the functioning of compound transmissions. A proper selection of the main ratio ranges do not require to presume anything about the power-split unit to be employed, but can optimize a specific use case, achieving the best matching between the loads and the adopted prime engine and CVU. A proper choice of the overall ratio spread A enhance the functioning of the engine, while A_i and A_o modulate the power flow through the CVU, which has a major impact on the transmission efficiency and on the power class of the CVU itself. Conversely, the initial speed ratios τ_{im} and τ_{om}, which scale the absolute speeds ω_i and ω_o, must be specified in order to make the CVU's shafts working in the desired absolute speed ratio ranges.

Our work is aimed to generality, so we provided in Section 3.1 some simple general guidelines to the choice of these ratio ranges, which can represent a valid starting point for the optimization process. Indeed, the proposed results minimize

p_i, thus being simple and general, but can be numerically refined after a first choice of the CVU by taking into account its real characteristics.

The same three ratio spreads A, A_i, and A_o govern a set of characteristic functions (Table 2) which guarantee the constructive feasibility of the planetary gear trains, let them work near their synchronism, and can suggest all the feasible layouts while representing all the power ratios between two shafts of each planetary gear train, being useful for both synthesis and analysis. Moreover, the graphical representation of these functions *(design-chart)* suggests the opportunities of constructive simplification, such as the adoption of planetary gear sets with the same number of teeth or the reduction of the strictly necessary ordinary gears.

As soon as the Willis' ratios and the layout, and thus the groups and the synchronous points, have been selected, the constructive constraints for the ordinary gears are defined too. The choice of their actual number and arrangement among the different solutions fulfilling the constraints of Table 3 can depend on several factors, but mostly on efficiency and constructive feasibility.

Assuming all the dissipation lumped inside the CVU is reliable in practice [57] (thanks to the high efficiency of ordinary and planetary gear sets, if the latter work near their synchronous conditions); however, a swift method in order to assess directly the power losses in the PSU will be provided in a future paper. Anyway, if the consequences of gear meshing losses are neglected at first, a later estimation of such power losses becomes very straightforward for both ordinary and planetary gearing, and it can be carried out by using one among the many model already in literature. For example, the systematic analysis approach proposed in [58] is well fitted for this purpose: known the layout, the speeds and the sign of ideal power flows, it offers enough constraints to solve the system of unknown real power flows.

6. Conclusions

This paper shows that the overall behavior of whatever PS-CVT does not depend on the constructive parameters, such as the characteristic speed ratio of the planetary gear trains, but mostly on three ratio spreads A, A_o, A_i.

These three ratio spreads can be assigned without knowing the final layout, which is later suggested by our method itself, among all the solutions (with one or two planetary gear trains and up to six ordinary gears) that optimize the CVU power flow. Indeed, the same three ratio spreads govern a set of characteristic functions whose graphical representation, named *design-chart*, permits a quick comparison between the main features of different feasible solutions, and can lead to important constructive simplifications, while taking efficiency-oriented choices.

Our method overturns the traditional design process, eliminating the need to have recourse to arbitrary or practice dictated choices. Its generality, simplicity and modularity appear well suited for further model development, such as the design of multi-mode PS-CVTs.

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