# Experimental results for a belt variator in transient conditions

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Keywords: rubber V-belt, motorcycle variator, transient behaviour, experimentation

SUMMARY. Several experimental results for the speed ratio shift of a small power rubber belt variator for motor-cycle application were collected with the aim at comparing with a recent theory developed by the authors on the transient mechanics of belt drives. Data for the wrap radius, angular speed and torque are recorded for each pulley during the tests and their numerical elaboration permits calculating the axial thrust on the primary and secondary pulleys according to previous models worked out by the authors for the pulley actuators, which are in particular of the loading spring type on the driven side and of the centrifugal mass type on the driver one. The theory on the variator transient mechanics yields a variety of quite different numerical solutions depending on the working conditions of the variator (shift speed, torque, etc.), but quite simple approximation formulas may be implemented for most of the operative range, giving a fine fit with the experimentation.

#### 1 INTRODUCTION

V-belt or chain transmissions with continuously variable speed ratio (CVT) are nowadays widespread in the automotive and motor-cycle application. In the first case, the nominal power is higher and metal chains or composite push belts of the Van Doorne type are used. In the second case the power is lower and the variable transmission is generally equipped with rubber belts reinforced by internal cords.

The usual working conditions of a CVT consist of sequences of transient phases where the wrap radii are continuously being varied on the one and the other pulley. It follows that a deep knowledge of the variator behaviour in condition of speed ratio shifting is very important. Likewise, a suitable collection of experimental data is indeed essential, with the aim at a useful comparison with theoretical models and a possible validation of their applicability.

The scientific literature offers quite few analyses of the shifting mechanical behaviour of a CVT (for example see [1,2]). One model has been recently proposed by our research group [3-5] and is grounded on a proper formulation of the mass conservation condition for the transient state, where the belt is considered as a continuum material flowing inside a stream tube formed by its own external surface, which is itself in motion during the ratio shift. Thus, the well known transport theorems can be applied, making reference for example to an elementary control dihedral between two meridian planes through the pulley axis.

In view of checking the validity of this model, a campaign of several experimental bench tests was carried out on a small power rubber belt variator for motorcycle application. The belt tensioning is achieved by a loading spring on the secondary pulley and the speed ratio is controlled by suitable centrifugal masses thrusting the primary pulley. Furthermore, the resistant torque corrects the secondary axial thrust by means of a helical coupling between the two half-pulleys. This makes the variator essentially automatic and then, the change of the speed ratio during the experimental tests may be obtained by modifying either the resistant torque or the driving speed.

Previous works of our research group have built up theoretical models for the actuator behaviour, which have been also experimentally validated. These models permit calculating the axial thrust on the one or the other pulley, in dependence on the speed ratio, the input speed and the output torque [6,7]. On the other hand, the total torque transferred to the belt and the axial thrust are strictly connected with the contact force distribution along the wrap arc. Therefore, by continuously measuring the torques, the rotation speeds and the winding radii during the drive shift, it is possible to estimate experimental correlation laws between the axial thrust and the shifting speed, to be compared with the theory.

The test bench includes a rubber belt variator, a DC electric motor, two torque-speed meters, two laser position sensors for the measure of the winding radii, some thermocouples and an electro-pneumatic disc brake. All devices are controlled by a feed-back software through a DAQ card.

- 2 THEORY
- 2.1 Full model

As the inertia forces due to the shift motion can be neglected in comparison with the other forces, the belt equilibrium conditions, in the directions tangential and normal to the belt remain substantially similar to the steady case. Moreover, similarly to the steady case, the radial penetration of each belt element is to be considered variable along the arc of contact due to the transverse elastic compression exerted by the pulley walls.

The conservation of mass on the contrary must be re-written taking into consideration the motion of the stream tube. With reference to Fig. 1, the total mass flux entering the dihedral control surface must be equal to the rate of mass increase inside the elementary control volume of width  $d\theta$  (which rate is zero in steady conditions). This leads to the closure of the mathematical model for the transient state, whose equations may be integrated by Runge-Kutta routines, starting from one of the endpoints of the arc of contact, as described in details in [3-5].



Figure 1. Scheme of belt element and control volume

While carrying out the numerical integration, a point of adhesion may happen to occur somewhere along the belt path, if the instant belt velocity becomes equal to the pulley velocity. Thereafter, an adhesive region develops, where the belt contact follows a spiral of Archimedes, which is inward or outward directed depending on the closing or opening motion of the half-pulley, so that the local belt motion takes place outwards or inwards due to the pulley rotation. Defining the dimensionless radial penetration as  $(r_{\infty} - r) / r_{\infty}$ , where  $r_{\infty}$  is the nominal winding radius for infinite stiffness of the belt in the transverse direction, its derivative with respect to the angular coordinate  $\theta$  turns out to be equal to the shift-totangential speed ratio,  $\dot{r}_{\infty}/(\omega r_{\infty})$ , along any possible adhesion region. Therefore, the belt trajectory must follow an inward spiral for a closing pulley ( $\dot{r}_{\infty} > 0$ ) and an outward spiral for an opening one ( $\dot{r}_{\infty} < 0$ ). Inside an adhesive region, a new variable shows up, as the adhesive friction force is not subject to the sliding friction laws, but a new condition must be added, i. e. the condition of zero relative velocity between the belt and the pulley. Clearly, an adhesive region must not spread beyond the attainment of the static friction limit, which indeed may be approached on decreasing the transverse compression along the spiral arc.

As a matter of fact, instead of real adhesion regions, just quasi-adhesive regions are encountered most frequently, where the belt path follows trajectories very close to the spiral curves mentioned above and the elastic penetration of the belt into the groove, which is on the other hand proportional to the axial thrust per unit angle of contact, shows a nearly linear trend.

#### 2.2 Approximate solutions

By the analysis of the numerical solutions of the full equations, some typical characteristics may be observed in general (see Fig. 2). This permits formulating approximate solutions, for driver or driven pulleys and for opening or closing half-pulleys, very useful for design purposes.



Figure 2. Example of numerical solutions for driver and driven pulleys. f = 0.45  $\alpha = 12.8^{\circ}$   $k = 2 \tan \alpha S_{\perp} / S_{//} = 0.15$ 

There are two very short seating and unseating regions at the entrance and the exit of the arc of contact. The belt penetration angle is positive in the first region, where the elastic penetration increases very rapidly, and negative in the second one, where it drops quickly to zero. The belt tension keeps nearly constant and the penetration angle gradient is relatively large and negative in both regions. These two boundary portions of the arc of contact may be ignored to a first approximation.

The remaining part of the wrap arc may be in theory divided into sliding and adhesive or quasi-adhesive regions, the last ones being generally larger or narrower than the first ones depending on the applied torque. Nevertheless, for realistic shifting speeds, as in the case of the present experimentation, full sliding occurs in practice along the whole arc of contact. Figure 2 shows two typical solution diagrams, for a driver and a driven pulley, obtained by a fourth-order Runge-Kutta integration of the full equations. As the shift speed is rather low, these curves are somehow similar to the steady case, but they are influenced by the parameter  $\dot{r}_{\infty}/(\omega r_{\infty})$  to a certain degree.

The diagram of the elastic penetration exhibits the nearly constant (quasi-adhesive) slope  $\partial(r_{\infty}-r)/\partial\theta \cong \dot{r}_{\infty}/\omega$  in the driver pulley case and, since the sliding angle  $\gamma$  (which is defined in Fig. 1) vanishes where the belt force attains roughly its maximum value, the penetration is given there by  $(r_{\infty} - r) / r_{\infty} \cong T_{\max} / [2S_{\perp}\tan(\alpha + \varphi)]$ , where  $T = F - qv^2$  indicates the "dynamic" belt tension, accounting for the belt centrifugal forces,  $S_{\perp} = 2\tan\alpha E_z h r_{\infty}^2/b$  is a transverse stiffness parameter of the belt (with the physical dimension of a force),  $E_z$  is the elastic modulus of the elastomer in the axial direction, *h* and *b* are the belt thickness and width and  $\alpha$  and  $\varphi$  are the groove angle and the friction angle [3-5]. Therefore, taking into consideration that the unit angle axial force  $F'_z$  is in practice equal to the quantity  $S_{\perp}(r_{\infty} - r) / r_{\infty}$ , the axial thrust on the driver side may be expressed in the ultimate form

$$F_{DR} \cong \frac{T_{\max} \cdot \Theta_{DR}}{2 \tan(\alpha + \varphi)} + \left(\frac{\dot{r}_{\infty} r_{\infty}}{\omega}\right)_{DR} \tan \alpha E_z \frac{h}{b} \Theta_{DR}^2$$
(1)

where  $\Theta_{DR}$  is the real wrap angle extent on the driver pulley, i. e. considering the contact arc reduction due to the belt flexural stiffness  $S_{f}$ . For both the driver and driven pulley, indicating with  $\Theta_{DR,DN(\text{geom.})}$  the geometric angle of contact in the hypothesis of ideal circular-straight belt path, the real angle of contact is given by

$$\Theta_{DR,DN} = \Theta_{DR,DN(\text{geom.})} - \frac{\sqrt{S_f}}{r_{DR,DN}} \left( \frac{1}{\sqrt{T_{\text{max.}}}} + \frac{1}{\sqrt{T_{\text{min.}}}} \right)$$
(2)

As regards the driven pulley, the radial penetration increases more than linearly, together with the belt tension force, in the final part of the wrap arc and moreover, the diagrams of the sliding angle  $\gamma$  and of the penetration angle  $\chi$  (defined in Fig. 1) indicate that  $\tan \gamma$  and  $\chi$  follow a nearly linear trend in the same region. Nevertheless, in order to get the simplest possible formulation, it is supposed that the axial thrust has an expression similar to Eq. (1), but replacing the subscript "max." with the one "min.", the subscript "DR" with "DN" and introducing a virtual angle of friction  $\psi$  in place of  $\varphi$ , where  $\psi$  is lower than  $\varphi$ . This angle is chosen in an empiric manner, grounding on the best fit with the

experimental results, and introduces a sort of correction for the ratio of the longitudinal deformation to the transverse deformation in order to account for the penetration increase in the second part of the wrap arc, which ratio is  $2 \tan(\alpha + \varphi) S_{\perp}/S_{\parallel}$  at the point of radial sliding ( $\gamma = 0$ ) ( $S_{\parallel}$  indicate the longitudinal stiffness of the belt) [3–5]. Then:

$$F_{DN} \cong \frac{T_{\min} \Theta_{DN}}{2 \tan(\alpha + \psi)} + \left(\frac{\dot{r}_{\infty} r_{\infty}}{\omega}\right)_{DN} \tan \alpha E_z \frac{h}{b} \Theta_{DN}^2$$
(3)

Another equation to be taken into consideration is given by the rotational equilibrium condition of the one or the other pulley. Indicating with  $C_{DR}$  the true effective torque exerted by the belt free spans on the driving pulley, one has

$$C_{DR} \cong r_{DR} \left( T_{\max} - T_{\min} \right) \tag{4}$$

The quantities  $F_{DR}$ ,  $F_{DN}$ ,  $r_{DR}$ ,  $r_{DN}$ ,  $C_{DR}$  and  $C_{DN}$  are measured during the experimental tests or calculated as will be described in the next sections, the geometrical characteristics of the variator are known and the material properties of the belt  $(S_{I/2}, S_{\perp}, S_f)$  can be estimated by the usual measuring procedures. Therefore, eliminating the angles of contact by use of Eq. (2), Equations (1) and (4) can be solved for the two unknowns,  $T_{max}$  and  $T_{min}$ , and then Equation (3) may be used either to calculate  $\psi$ , or to verify the goodness of some presumed value of  $\psi$ , for example constant for the whole test.

#### **3** ACTUATOR MODEL

The forcing devices for the driver and driven pulleys are shown in Fig. 3 and the analysis of their behaviour is widely described in [7].



Figure 3. Belt forcing devices on the driver and driven sides

On the driver side, a centrifugal mass system permits controlling the speed ratio by means of the driving shaft speed. The measure of the fixed ramp angle and an optical experimental survey of the concave surface inside the movable cap yield the relationship for the axial thrust in dependence on the half-pulley axial position (speed ratio) and on the angular speed:

$$F_{DR} \cong \frac{M\omega_{DR}^2 r_M}{2\tan\alpha} \frac{dr_M}{dr_{DR}}$$
(5)

where  $r_M$  is the radial position of the centrifugal mass centres, M the total mass and  $dr_M/dr_{DR}$  can be calculated in dependence on  $r_{DR}$  and on the ramp shape, as shown in [7].

As regards the driven pulley actuator, a loading spring produces the belt forcing and it is remarkable that a torque correction is exerted on the axial thrust due to the helical shape of the coupling tracks between the movable and fixed half-pulleys. The total axial thrust is [7]:

$$F_{DN} \cong F_0 + K\Delta + \frac{C_{DN}}{d} \tan \beta$$
(6)

where  $F_0$  is the pre-load, K the spring stiffness,  $\Delta$  the spring deflection,  $\beta$  and d the helix angle and diameter.

The roller, spring and belt parameters are the ones of Table 1 and Figure 2, while the actuator geometry is the same of ref. [7].

Roller		Belt		Spring	
Diameter	16 mm	Length	758 mm	$F_{0}$	300 N
Total mass	46.6 gr	$S_f$	$7.3 kN*mm^2$	K	5.42 N/mm
		<i>S</i> //	120 kN		
Table 1					

Table 1

Summing up, the axial load on the one and the other pulley is not measured directly, but is calculated by measure of the angular speed, speed ratio, wrap radii and torques according to previous models worked out by the authors which have been experimentally validated [7].

#### 4 TEST BENCH

The test bench is the same of ref. [7] and is briefly described hereafter.

As shown in Fig. 4, a DC electric motor drives the transmission, while a disc brake, which is pneumatically operated, exerts the output torque. The CVU is an automatic variator of the motor-cycle type with a downstream gears coupling with ratio 13 and a centrifugal clutch. Owing to some speed restrictions of the DC electric motor, another gear coupling was connected upstream of the variator, stepping up the speed with ratio reciprocal of downstream gears. Moreover, the centrifugal clutch was blocked.

The driver and driven speed and torque are measured by two speed-torque meters of the strain-gauge type while the belt pitch radii are measured by two LASER displacement sensors. Of course, all those data that are taken while one of the movable half-pulleys (in practice, only the driver half-pulley in the present variator) stands still at its end stops are not considered for the calculation of the actuator axial thrust. The temperature of the gearbox oil is picked up by means of two thermocouples. All signals are conveyed to a data acquisition system with a sampling frequency of 20 Hz and analyzed by a proper routine. The experimental data are then processed by a low-pass Butterworth filter of the first order with phase lag retrieval, whose cut-off frequency is equal to the fundamental harmonic, such as may be determined by FFT.

The power losses due to the upstream and downstream gears and to the belt stiffness are taken into consideration in the evaluation of the true effective torque exerted by the belt free spans on the driven pulley. In practice, assuming equal losses for both sides of the variator, the torque measured by the output-torque meter is divided by  $13\sqrt{\eta}$ , where  $\eta$  is the instant variator efficiency.



Figure 4. Test bench

### 5 RESULTS

A considerable number of experimental tests were carried out, for several loading torques on the driven shaft and various input speed shift rate.

As examples, Figures 5 to 13 show the results of several shift up tests, where the driver angular speed change rates and the constant resistant torques are indicated in the captions. In the figures red, blue and black lines are respectively for driven, driver and equation (3) while the arrows indicate the reference scale (left or right) for the curves.



Figure 7. Shift test 3 ( $\dot{n}_{DR}$  = 520 rpm/s  $C_2$  = 40 Nm)



Figure 10. Shift test 6 ( $\dot{n}_{DR}$  = 780 rpm/s  $C_2$  = 60 Nm)



Figure 13. Shift test 9 ( $\dot{n}_{DR} = 780 \text{ rpm/s}$   $C_2 = 80 \text{ Nm}$ )

The measured-calculated axial thrust shows an increasing trend with the time, both on the driver and driven side. The starting and ending phases, in which the driver half-pulley is in contact with one of its end stops and is subject to the consequent constraining force, were all ignored in the elaboration of the results.

A remarkable result is that the angle  $\psi$  introduced by Eq. (3) may be roughly considered constant with a fine approximation throughout each single test at constant shift speed and may vary, but quite little, on changing the operative conditions, that is the shift

speed and the applied load. Nevertheless, very good results can be obtained by simply choosing an average value for this angle, as clearly shown in Figures 5-13, where the angle  $\psi$  was put equal to the mean value 0.86 $\varphi$  once for all when calculating the "theoretical" axial thrust on the driven side by Eq. (3). This approximation is also justified by Fig. 14, where the optimal ratio  $\psi/\varphi$  is plotted as a function of the input speed shift rate, for different values of the applied output torque, and may be seen contained inside a very narrow numerical band. In each of the Figure 5 to 13, the diagram on the right shows the change of the winding radii of the driver and driven pulleys, together with the shift speed rates  $\dot{r}_{\alpha}$ , to give an idea of the test execution modality.

It is believed that the narrow strip spanning the variation of the ratio  $\psi / \varphi$  is characteristic and typical for each single variator, which hypothesis should have to be verified by carrying out a large campaign of tests on other belt drives. The collection of these averaged parameter values for a large class of variable speed drives could provide a very simple tool for their design and for the analysis of their expected behaviour.



Figure 14. Optimal ratio of angle  $\psi$  to friction angle  $\varphi$  versus driver shift speed, for various output torques.

### 6 CONCLUSION

The results of several experimental tests on the mechanical behaviour of a motor-cycle automatic variator during the speed ratio change have been collected and presented. In particular, such tests were carried out with different but constant output torque and input speed shift rate.

The theoretical model for the transient behaviour of V-belt drives, formerly elaborated by the authors, would indeed produce many different numerical solutions in dependence on the different operating conditions. Nevertheless, a very simple formulation to approximate those theoretical results was here proposed as regards in particular the axial thrust on both the driver and driven pulley.

On the other hand, previous works of our research group have built up a theoretical model which permits calculating the actuator axial thrust on the one or the other pulley, in dependence on the speed ratio, the input speed and the output torque.

The combined use of the two models gives a fine fit with the experiments and since the knowledge of the axial thrust is fundamental in designing this type of variators, this formulation may turn out to be very helpful for the transmission engineers. Moreover, it is believed that the presented transient axial thrust model should be applicable also to rubber V-belt variators of different type and with other regulation devices.

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In view of checking the validity of this new model, a campaign of several experimental bench tests was carried out on a rubber belt variator for motorcycle application. The belt tensioning is achieved by a loading spring on the secondary pulley and the speed ratio is controlled by suitable centrifugal masses thrusting the primary pulley. Furthermore, the resistant torque corrects the secondary axial thrust by means of a helical coupling between the two half-pulleys. This makes the variator essentially automatic and then, the change of the speed ratio during the experimental tests may be obtained by modifying either the resistant torque or the driving speed. The test bench includes a DC electric motor, two torque-speed meters, two laser position sensors for the measure of the winding radii, some thermocouples and an electro-pneumatic disc brake. All devices are controlled by a feedback software through a DAQ card. The axial load on the one and the other pulley is not measured directly, but is calculated by measure of the angular speed, speed ratio, wrap radii and torques according to previous models worked out by our research group which have been experimentally validated [7]. Therefore, by continuously measuring the torques, the rotation speeds and the winding radii during the drive shift, it is possible to estimate experimental correlation laws between the axial thrust and the shifting speed, to be compared with the transient V-belt theory.

The results of several experimental tests have been collected and presented. In particular, such tests were carried out with different but constant output torque and input speed shift rate. The comparison between theory and experiments has shown a fine fit and since the knowledge of the axial thrust is fundamental in designing variators of the type tested here, the present formulation for the rubber V-belt axial thrust during the speed ratio shift, combined with the actuator axial thrust model of ref. [7], may turn out to be very helpful for the transmission engineers.

Moreover, it is believed that the presented transient axial thrust model should be applicable also for rubber V-belt variators of different type and with other regulation devices.

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