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Efficiency of Split-Way CVT's. A Simplified Model.

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ABSTRACT

In this paper, a general formula is obtained for the efficiency of all types of Split-Way CVT, consisting of one or two epicyclic trains, a number of fixed ratio gears and one variable speed unit (CVU). It is based on the application of the principle of virtual works and on the simplifying hypothesis that all the dissipation is lumped inside the CVU.

The comparison with some experimental results from the literature shows a surprisingly good fit, also in the neighbourhoods of zero speed ratio.

INTRODUCTION

The possibility of changing the speed ratio continuously between the primary and the secondary shaft of a mechanical transmission is a very interesting topic, mainly in the automotive field.

With the aim at overcoming the main drawbacks of mechanical variators, in terms of transmissible power, width of the speed ratio range and efficiency, the idea of using the continuously variable unit (CVU) combined with a differential drive has stimulated the researchers in the last two decades.

Although the names of this kind of mechanical transmissions are somewhat different in the literature (Split-Way CVT's, Split Power CVT's, Infinitely Variable Transmission IVT, Split Path Transmission, et cetera), they consist always of one or two epicyclic trains, a number of fixed ratio gears and one CVU. Broadly speaking, they can either amplify the overall speed ratio range in comparison with the simple variator, though entailing a higher variator power ratio (i.e. power recirculation in the CVU) and a lower efficiency, or can be designed to get opposite features. Anyway, the combination of a large speed range and a low variator power ratio, with a higher transmission efficiency, can be obtained by using several modes (that is associating two or more split way layouts which commute to each other by brakes or clutches).

There are numerous examples of Split-Way CVT (SW-CVT) in literature, also of the multi mode type, but they use only one epicyclic train in general (for example [1] [2] [3] [4] [5], as well as many others). On the contrary, applications with two epicyclic trains are rare,

though, the use of a second epicyclic train can improve the variator power ratio substantially, as shown in references [6][7][8][9].

In any case, the theoretical models proposed to describe the kinematical behaviour and to evaluate the performances in terms of power flows and efficiency, are based in general on the knowledge of the structural characteristics of each transmission unit.

Nevertheless, assuming ideal working, it has been recently proved, that the overall behaviour of these drives, either with one or two epicyclic trains, depends only on three functional parameters, which can be defined regardless of the drive layout and of its structural components [10]. These three parameters, also named apertures A, A_{ν} and A_i represent respectively the ranges of

- the speed ratio of the whole CVT transmission;
- the speed ratio of the internal CVU;
- the angular speed of the CVU primary shaft.

Once fixed such three parameters, there is only one possible trend for the power flow, the input angular speed and the speed ratio of the CVU, but it is possible to traduce into practical realization a great variety of solutions that, though structurally different from each other, are functionally equivalent at all and foremost, they are "seen" by the inside CVU in the same way.

This innovative black boxes model changes radically the approach to the Split-Way CVT and offers a powerful tool for their design, but it suffers the hypothesis of absence of losses.

This paper integrates the model presented in [10] and shows that the suggested framework remains valid also in real working conditions if one considers all the dissipation lumped inside the CVU. This simplifying hypothesis is supported by the well-known fact that welldesigned CVU's generate generally larger loss than welldesigned epicyclic or fixed trains.

On the basis of this hypothesis and of the principle of virtual works, a general formula is obtained for the efficiency, valid for whichever multi-way CVT drive. This efficiency formula still depends on the three forementioned functional parameters and, obviously, on the CVU efficiency. The comparison with some experimental results from the literature shows a surprisingly good fit, also for drives extending the speed ratio range down to zero or to reverse motion. Consequently, it can be used for a very fast estimation of the overall efficiency of whichever Split-Way CVT's,

especially at an early stage of the design process when the drive is not yet defined in details.

FUNCTIONAL BEHAVIOUR OF SPLIT WAY CVT'S

In this section, a short review of the mathematical model presented in [10] is reported.

Every multi-way CVT drive can be thought as a black box with a CVU inside (Figure 1).



Figure 1 SW-CVT as a black box

Indicating with n... the angular speed of the shafts, one has:

$t = \frac{n_{OUT}}{n_{IN}}$	overall speed ratio	(1)
$t_v = \frac{n_o}{n_i}$	CVU speed ratio	(2)
$t_i = \frac{n_i}{n_{IN}}$	dimensionless input angular speed of CVU	(3)

Moreover, indicating with the subscript $(.)_{M}$ the values assumed by all variables in overdrive condition (when |t| is maximum) and with the subscript $(.)_{m}$ in underdrive

(when |t| is minimum), the apertures A, A_v and A_i are defined by:

$$A = \frac{t_{M}}{t_{m}} \qquad A_{v} = \frac{t_{v_{M}}}{t_{v_{m}}} \qquad A_{i} = \frac{t_{i_{M}}}{t_{i_{m}}}$$
(4)

In ideal working conditions, the following formulas may be proved:

$$p_{v} = \frac{\left[(A - A_{i}A_{v}) + \frac{t}{t_{m}} (A_{i}A_{v} - 1) \right] (A - A_{i}) + \frac{t}{t_{m}} (A_{i} - 1)}{\frac{t}{t_{m}} A_{i} (A_{v} - 1)(A - 1)}$$
(5)
$$(A - A_{i}A_{v}) + \frac{t}{t_{m}} (A_{i}A_{v} - 1)$$

$$\frac{t_{v}}{t_{v_{m}}} = \frac{t_{m}}{(A-A_{i}) + \frac{t}{t_{m}}(A_{i}-1)}$$
(6)
$$\frac{t_{i}}{t_{i_{m}}} = \frac{(A-A_{i})}{(A-1)} + \frac{(A_{i}-1)}{(A-1)}\frac{t}{t_{m}}$$
(7)

where $p_v = P_i / P_{IN}$ is the CVU power ratio.

So, the functional behaviour of whichever split-way CVT turns out to be completely defined only by the above three apertures. In particular, the CVU will be always subject to the same boundary conditions as regards its own speed ratio t_{v_i} power p_{v_i} , angular speed t_i and input torque $c_v = p_v/t_i$.

In a preliminary design of a split-way CVT, the task to be fulfilled is: to realize a continuously variable drive with an expected speed ratio range (aperture A) by means of an assigned variable speed unit (aperture A_v), trying in the meanwhile to limit the power flow through the variable speed ratio way as much as possible.

Well, the quality of the result does not depend on the drive scheme, nor on the structural components of the transmission, except in so far as they may influence the aperture A_i of the CVU input shaft as described in section 3 of [10]. Briefly: for a given drive, there is only one value of A_i , while, fixing A_i , several schemes and a myriad of mechanisms may be realized, all structurally different. Moreover, A_i is effectively an independent variable if inside the black box of Figure 1 there are two epicyclic trains, while it is univocally constrained by A and A_v in case of a single epicyclic train.

EFFICIENCY OF SPLIT-WAY CVT'S

Considering now the SW-CVT of Figure 1 in its real operation with friction losses, one has

$P_{IN} = P_{IN}$	input power	(8)
_		

$$h = \frac{P_{OUT}}{P_{IN}}$$
 overall efficiency (9)
$$h_{\nu} = \frac{\overline{P}_{o}}{\overline{P}_{i}}$$
 CVU efficiency (10)

where over scored symbols indicate real operation. Suppose now to take away the CVU from the SW-CVT of Figure 1 and leave the torque applied at the shaft stumps as external reaction torque as in Figure 2.

The obtained mechanical system has two degree of freedom, and the angular speed of whatever shaft can be expressed as a linear combination of any two of the others, so that

$$t_o = \frac{n_o}{n_{IN}} = a_o + b_o t \tag{11}$$

is the dimensionless output angular speed of the CVU. The dimensionless constants a_o and b_o can be expressed as functions of the three apertures above defined (see [10] for more details).

$$a_{o} = t_{i_{m}} t_{v_{m}} \frac{A - A_{i} A_{v}}{A - 1}$$
(12)

$$b_o = \frac{t_{v_m} t_{i_m}}{t_m} \frac{A_i A_v - 1}{A - 1}$$
(13)

Moreover, if one considers all the dissipation lumped inside the CVU, the system of Figure 2 results a mechanical system without dissipation and therefore it is possible to apply the principle of virtual works.



Figure 2 Torque applied at the shaft stumps

This principle requires the vanishing of the total work done by the applied torque $\overline{C}_{...}$ for any virtual rotation

 dJ_{\dots} of the shafts allowed by the system constraints

 $C_{IN}dJ_{IN} - \overline{C}_{OUT}dJ_{OUT} = \overline{C}_i dJ_i - \overline{C}_o dJ_o$ (14) Moreover, due to the previous hypothesis, the

equilibrium configuration implies:

$$\Pi dt = C_{IN} J_{IN} - C_{OUT} J_{OUT} = C_i J_i - C_o J_o$$
(15)
or the equivalent relationship
$$\Pi = (1 - h) P_{IN} = (1 - h_v) \overline{P_i}$$
(16)

where the quantities J = n dt are real shaft rotations,

such that $J_{OUT}/J_{IN} = t$, $J_o/J_i = t_v$, while Π indicates the overall power losses.

Combining all the previous equations, one gets at last (see the appendix for details)

$$h = 1 - \left[\frac{1}{1 - \frac{A_i A_v - 1}{A_i A_v - A} \frac{t}{t_m}} + \frac{1}{(1 - h_v) p_v} \right]^{-1}$$
(17)

where p_v is the CVU power ratio in ideal working condition, given by equation (5).

In order for equation (17) to be correct, the power through the CVU has to flow effectively from shaft "i" toward "o" ($p_v>0$), otherwise h_v has to be changed into $1/h_v$. It is remarkable that, according to the adopted agreements, p_v is always positive in all the drive schemes of reference [10], except for the Split-Way CVT's of the reverse-forward type ($A\leq$ -1), where $p_v>0$ for forward speed and $p_v<0$ for reverse speed.

Once known the overall efficiency, the calculation of the CVU power ratio \overline{p}_{v} in real working condition is quite easy:

$$\overline{p}_{v} = \frac{\overline{P}_{i}}{P_{IN}} = \frac{1-h}{1-h_{v}} = \left[\frac{(1-h_{v})}{1-\frac{A_{i}A_{v}-1}{A_{i}A_{v}-A}\frac{t}{t_{m}}} + \frac{1}{p_{v}}\right]^{-1} (18)$$

as well as the real input torque

$$\overline{c}_{v} = \overline{p}_{v} / t_{i} \tag{19}$$

We draw thus the conclusion that, under the assumed hypothesis, the framework of the SW-CVT general

model presented in [10] remains valid also in real working condition.

COMPARISON WITH EXPERIMENTAL DATA

In this section, the results by the previous formulas are compared with three set of experimental data found in literature [11][12][13].

This requires the preliminary calculation of the three apertures A, A_{ν} , A_{i} , by means of the structural characteristics of each test bench, following the procedure of Section 5, *Guidelines to Analyse*, of [10].

Figure 3 represents the test bench used in [11]. It is arranged with one epicyclic train E.T. and one fixed ratio coupling (in general reference [10] indicates by C.E.T. the group E.T. plus fixed ratio couplings).



Figure 3 Test bench of reference [11]

According to the classification of reference [10] one has: equivalent drive scheme "A", C.E.T.2 whit only fixed ratio coupling k_c , while the internal connections of the E.T.2 correspond to the combination 2b (see Figure 4).



Figure 4 Scheme and data of the drive in Figure 3

Consequently, by use of the formulary of [10], one has t_{vm} = 0.41, A_v =5.244 $\Rightarrow t_{vM}$ =2.15 t_m = -0.18, A= -18.33 $\Rightarrow t_M$ =3.3 t_{im} = 1 , A_{im} =1

The drive appears of the reverse-forward type ($A \le -1$), and then $p_v > 0$ for forward speed and $p_v < 0$ for reverse speed.

Figures 5-7 show the comparison between the theoretical efficiency obtainable by equation (17) and the experimental data in [11]. The efficiency $h_v(t)$ is calculated for different load conditions, interpolating the experimental results for the CVU efficiency reported in [12].



Figure 5 Comparison between equation (17) and the experimental data of [11] (full load).

Figure 8 shows a similar comparison for the CVU power ratio (the absolute values of the reciprocals $|\overline{p}_{\nu}|^{-1}$ are reported in the diagram). The experimental point (0,0) of Figure 8 has no sense in actual fact, because it implies the concept of no dissipation and obviously, this is not possible in an experiment. It is very probably due to a typo of [11].



Figure 6 Comparison between equation (17) and the experimental data of [11] (mid load).



Figure 7 Comparison between equation (17) and the experimental data of [11] (low load).



Figure 8 Comparison between equation (18) and the experimental data of [11] (full load).

Figure 9 represents the test bench used in [12]. It is similar to the previous one but with the motor mounted on another shaft.



Figure 9 Test bench of reference [12]

According to the classification of reference [10] one has: equivalent drive scheme "A b", C.E.T.2 whit only fixed ratio coupling k_c , while the internal connections of the E.T.2 correspond to the combination 2b. (see Figure 10).



Figure 10 Scheme and data of the drive in Figure 9

Consequently, by use of the formulary of [10], one has t_{vm} =0.446, A_v =4.48 \Rightarrow t_{vw} =2

$$t_m$$
=-0.242, A=-6.192 \Rightarrow t_m =1.5

 $t_{im} = 1/t_{vm}$, $A = 1/A_v$

Again, the drive appears of the reverse-forward type (A \leq -1) and then $p_v>0$ for forward speed and $p_v<0$ for reverse speed.

Figures 11-12 show the comparison between the theoretical efficiency by equation (17) and the experimental data. The efficiency $h_v(t)$ is calculated, for different load conditions, interpolating the experimental results for the CVU efficiency reported in [12].



Figure 11 Comparison between equation (17) and the experimental data of [12] (full load).



Figure 12 Comparison between equation (17) and the experimental data of [12] (mid load).

Figure 13 shows the same comparison for the CVU power ratio (also here, the absolute values of the reciprocals $|\overline{p}_{v}|^{-1}$ are reported in the figure).



Figure 13 Comparison between equation (18) and the experimental data of [12] (full load).

Figure 14 represents the test bench used in [13].

It is arranged with two epicyclic trains and planned for two-mode operation. The complete rig is used in the I mode, while in the II mode, the epicyclic train 2 is blocked and the coupling k_c is disengage by a suitable switching device.



Figure 14 Test bench of reference [13]

According to the classification of reference [10], the equivalent drive scheme of the I mode is "III b", the C.E.T.1 has only the fixed ratio coupling with ratio k_b while the internal connections of the E.T.1 are the ones corresponding to the combination 3. The C.E.T.2 has not the reducer k_{OUT} while the internal connections of the

E.T.2 correspond to the combination 3 b (see Figure 15).

Consequently, by use of the formulary of [10], one has t_{vm} =1, A_v =3.6 \Rightarrow t_{vM} =3.6 t_m =-0.403, A=-0.792 \Rightarrow t_M =0.319 t_{im} =1.107, A_i =0.204 \Rightarrow t_{iM} =0.226



Figure 15 Schemes and data of the drive in Figure 14

The I mode appears to be of the reverse-forward type (A \leq -1) and $p_{v}>0$ for forward speed and $p_{v}<0$ for reverse speed.

The equivalent drive scheme of the II mode is "B". The C.E.T.1 is the same of the I mode, while on the branch OUT, there is a fixed ratio coupling with ratio k_d .

The change of mode implies another inversion of the power flux through the CVU and consequently

 $t_{v_M | \text{Imode}} \cdot t_{v_m | \text{IImode}} = t_{v_M | \text{IImode}} \cdot t_{v_m | \text{Imode}} = 1.$

By use of the formulary of [10], one has

 t_{vm} =1/3.6, A_v =3.6 \Rightarrow t_{vm} =1

 t_m =0.319, *A*=4.895 \Rightarrow t_M =1.563

 $t_{im} = t_m/t_{vm}, A_i = A/A_v$

Note that $t_M |_{I \mod} = t_m |_{II \mod}$, therefore the switch from the first mode to the second one occurs in rotational synchronism of the connecting/disconnecting shafts. The overall aperture of the two mode is A=-3.877.

Figure 16 shows the comparison between the theoretical efficiency obtainable by equation (17) (continuous line) and the experimental data (points). The efficiency $h_v(t)$ is calculated interpolating the experimental results for the CVU efficiency reported in [13].



Figure 16 Comparison between equation (17) and the experimental data of [13]

The abrupt change of the efficiency when switching from one mode to the other is due to the sudden increase/decrease of the power fraction through the variator.

Figure 17 shows the comparison for the CVU power ratio (in the figure, the absolute value $|\overline{p}_v|$ is reported).



Figure 17 Comparison between equation (18) and the experimental data of [13].

All the comparisons show a surprisingly good fit between the theory and the experiments, especially if one consider the uniqueness and the simplicity of the proposed formula.

In order to achieve a greater precision, the loss inside the epicyclic gearing should be considered too. Although the denominations and the procedures to derive these losses are somewhat different in the literature (for example [14] [15]), the adopted concept is always that of the "apparent" power: the lost power depends on the torque and on the angular velocity relative to the carrier and care must be taken to determine whether the "apparent" power sign changes while varying the overall speed ratio. This analysis turns out to be very tedious to carry out, particularly if there are two epicyclic trains in the drive. Moreover, it leads to a sensible improvement of the global efficiency estimation of the Split-Way CVT, only if the epicyclic trains work far from their synchronous condition (same speed of the three shafts), due to the relatively high meshing speed of the teeth. Anyway, these methods are not implementable in a general black box model, since a precise information on the inside structure of each epicyclic gearing is needed.

CONCLUSION

In this paper a unique general formula for the efficiency of Split-Way CVT's is presented. It was obtained by means of the principle of virtual work under the simplifying hypothesis that all the dissipation is lumped inside the CVU. The comparison with some experimental results from the literature shows a surprisingly good fit, also in the neighbourhoods of zero speed ratio.

Moreover, this formula integrates the innovative black boxes model for split-way CVT's presented in [10], extending its applicability to real running with power dissipation. The mentioned model changes the approach to this type of mechanical transmission radically, because it proves that their functional behaviour, as well as the CVU boundary conditions, depend only on three functional parameters which can be defined once and for all, regardless of the drive layout and of its structural components.

Therefore, the efficiency formula of this paper and the model of reference [10] offer a very powerful tool for the design of whichever split-way CVT, especially at an early stage of the design process, because they permit to optimize the functional behaviour of the transmission even before knowing how it will be effectively realized.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

Notation

- *A* overall aperture (Equation 4)
- *a*_o dimensionless constant
- A... aperture (Equation 4)
- *b*_o dimensionless constant
- *c*_v CVU torque ratio

- C... Torque
- k... ratio of fixed ratio coupling
- n... angular speed
- p_v CVU power ratio
- P... power
- g_1 E.T.1 characteristic torque ratio
- *g*₂ E.T.2 characteristic torque ratio
- $J_{\rm m}$ real shaft rotation
- $dJ_{\rm m}$ virtual shaft rotation
- h overall efficiency
- *h*_v CVU efficiency
- t overall speed ratio
- t... speed ratio
- *y*₁ E.T.1 characteristic speed ratio
- y₂ E.T.2 characteristic speed ratio
- P overall power losses
- ... Over scored = real operation

Subscript

- a drive branch
- *b* drive branch
- c drive branch
- d drive branch
- *i* CVU input shaft
- IN drive input shaft
- *m* underdrive condition
- M overdrive condition
- o CVU output shaft
- OUT drive output shaft
- v CVU

APPENDIX

Summing equations (14) and (15) one gets:

$$C_{IN}J_{IN}\left(1+\frac{dJ_{IN}}{J_{IN}}\right) - \overline{C}_{OUT}J_{OUT}\left(1+\frac{dJ_{OUT}}{J_{OUT}}\right) =$$

$$= \overline{C}_{i}J_{i}\left(1+\frac{dJ_{i}}{J_{i}}\right) - \overline{C}_{o}J_{o}\left(1+\frac{dJ_{o}}{J_{o}}\right)$$
(20)

Since $\overline{C}_{...} \cdot J_{...} = \overline{P}_{...} \cdot \Delta t$, considering equations (9) (10) one gets

$$P_{IN}\left[\left(1+\frac{dJ_{IN}}{J_{IN}}\right)-h\left(1+\frac{dJ_{OUT}}{J_{OUT}}\right)\right] =$$

$$=\overline{P}_{i}\left[\left(1+\frac{dJ_{i}}{J_{i}}\right)-h_{v}\left(1+\frac{dJ_{o}}{J_{o}}\right)\right]$$
(21)

whence $\overline{\mathbf{n}}$

$$\overline{p}_{v} = \frac{P_{i}}{P_{IN}} = \frac{(1-h) + \left(\frac{dJ_{IN}}{J_{IN}} - \frac{dJ_{OUT}}{J_{OUT}}\right) + \left(\frac{dJ_{OUT}}{J_{OUT}} - h\frac{dJ_{OUT}}{J_{OUT}}\right)}{(1-h_{v}) + \left(\frac{dJ_{i}}{J_{i}} - \frac{dJ_{o}}{J_{o}}\right) + \left(\frac{dJ_{o}}{J_{o}} - h_{v}\frac{dJ_{o}}{J_{o}}\right)}$$
(22)
Since

Since

$$\left(\frac{dJ_{OUT}}{J_{OUT}} - \frac{dJ_{IN}}{J_{IN}}\right) = \frac{\partial t}{t}$$
(23)
$$\left(\frac{dI}{dI} - \frac{dI}{dI}\right) = \frac{\partial t}{dt}$$

$$\left(\frac{dJ_o}{J_o} - \frac{dJ_i}{J_i}\right) = \frac{\partial t_v}{t_v}$$
(24)

replacing equations (23) and (24) into (22) and considering (16) one gets

$$\frac{(1-h)\left(1+\frac{dJ_{OUT}}{J_{OUT}}\right)-\frac{dt}{t}}{(1-h_{v})\left(1+\frac{dJ_{o}}{J_{o}}\right)-\frac{dt_{v}}{t_{v}}} = \frac{(1-h)}{(1-h_{v})}$$
(25)

whence

$$(1-h)\left(1+\frac{dJ_{OUT}}{J_{OUT}}\right) - \frac{dt}{t} =$$

$$= (1-h)\left[\left(1+\frac{dJ_o}{J_o}\right) - \frac{1}{(1-h_v)}\frac{dt_v}{t_v}\right]$$
and
$$(26)$$

$$\left(1-h\right)\left[\left(\frac{dJ_{OUT}}{J_{OUT}}-\frac{dJ_o}{J_o}\right)+\frac{1}{\left(1-h_v\right)}\frac{dt_v}{t_v}\right]=\frac{dt}{t}$$
But
$$(27)$$

$$\left(\frac{dJ_{OUT}}{J_{OUT}} - \frac{dJ_o}{J_o}\right) = \left(\frac{dJ_{OUT}}{J_{OUT}} - \frac{dJ_{IN}}{J_{IN}}\right) + \left(\frac{dJ_{IN}}{J_{IN}} - \frac{dJ_o}{J_o}\right) = \frac{dt}{t} - \frac{dt_o}{t_o}$$
(28)
where

w

$$t_{o} = \frac{J_{o}}{J_{IN}} = \frac{n_{o}}{n_{IN}} \text{ (see equation (11)).}$$

and since in ideal working condition one has [10]
$$p_{v} = \frac{\frac{dt}{t}}{\frac{dt}{t_{v}}}$$
(29)

 $\frac{t_v}{dt_v}$ replacing equation (28) into (27) and multiplying by one gets

$$(1-h) \left[p_{v} - \frac{dt_{o}}{t_{o}} \frac{t_{v}}{dt_{v}} + \frac{1}{(1-h_{v})} \right] =$$

$$= (1-h) \left[p_{v} - \frac{dt_{o}}{t_{o}} \frac{t}{dt} P_{v} + \frac{1}{(1-h_{v})} \right] = p_{v}$$
(30)
whence

whence

$$\frac{1}{1-h} = 1 - \frac{t}{t_o} \frac{dt_o}{dt} + \frac{1}{(1-h_v)p_v}$$
(31)

that is, by equation (11),

$$\frac{1}{1-h} = \frac{a_o}{a_o + b_o t} + \frac{1}{(1-h_v)p_v}$$
(32)

At last, replacing equations (12) and (13) into (32), one gets 1 1

$$\frac{1}{1-h} = \frac{1}{1-\frac{A_i A_v - 1}{A_i A_v - A} \frac{t}{t_m}} + \frac{1}{(1-h_v)p_v}$$
(33)

which is exactly equation (17).