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VISCOELASTIC RESPONSE OF RUBBER BELTS

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Prove di carico su cinghie di trasmissione commerciali con corpo di gomma e corda interna in materiale sintetico evidenziano fenomeni d'elasticità ritardata.

E' noto che le proprietà meccaniche e reologiche dei polimeri dipendono dal processo di preparazione e che è possibile aumentare o ridurre l'attrito interno entro larghi limiti in dipendenza delle caratteristiche richieste. Per quanto riguarda le cinghie di trasmissione, non è opportuno aumentare la viscosità propria del corpo di gomma, poiché così si esaltano le perdite per isteresi nel ciclico cambiamento di curvatura durante le fasi d'avvolgimento e svolgimento dalla puleggia. Può essere invece conveniente incrementare l'isteresi nella corda interna, giacché il ritardo viscoso delle deformazioni elastiche longitudinali riduce la velocità di strisciamento sulla puleggia, con conseguenti vantaggi riguardo all'usura della cinghia e, in piccola parte, anche al rendimento.

Il presente approccio analizza il comportamento viscoelastico delle cinghie nelle trasmissioni di potenza, formulandone l'equazione costitutiva in base ad un modello del tipo elemento Maxwell con molla in parallelo ed individuando le differenze rispetto al modello elastico convenzionale.

Si mostrano inoltre i risultati di prove di carico-scarico, "creep" e "relaxation" su alcune cinghie, dando un'interpretazione teorica dei diagrammi sperimentali ottenuti.

VISCOELASTIC RESPONSE OF RUBBER BELTS

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SOMMARIO

Si analizza il comportamento viscoelastico delle cinghie nelle trasmissioni di potenza, formulandone l'equazione costitutiva in base ad un modello a tre parametri del tipo elemento Maxwell con molla in parallelo. Si evidenziano gli effetti del ritardo viscoelastico della deformazione longitudinale sullo scorrimento della trasmissione e quindi i possibili vantaggi in termini d'usura e di rendimento. Si mostrano infine risultati sperimentali di prove di carico-scarico, "creep" e "relaxation" su alcune cinghie di tipo commerciale.

ABSTRACT

The viscoelastic behavior of rubber belts is analyzed using a three-parameter constitutive model based on the Maxwell element plus a shunt spring. The research aims at the decrease of the belt slip on the pulley by a retard of the longitudinal elastic response, to the advantage of the belt wear and the efficiency. Experimental results from load, unload, creep and relaxation tests on several belts by a tension testing machine are also shown.

1. INTRODUCTION

Load tests on current rubber belts reinforced by internal synthetic chords give clear evidence of retarded elasticity phenomena.

It is well known that the viscoelastic properties of polymers depend on the manufacturing process and it is possible to increase or reduce the internal friction in dependence on the requested working characteristics [1, 2, 3, 4].

As to traction belts, an increase of the body rubber viscosity is not convenient, due to the bending hysteresis losses when unseating and seating on the pulley. On the contrary, an increase of the internal chord hysteresis may yield some advantage, as the belt slip may be reduced by the retarded longitudinal deformation.

The present analysis deals with the mechanics of viscoelastic belts assuming a three-parameter constitutive model for the belt element and compares with the pure elastic behavior.

Some experimental results from tensile tests on rubber V-belts show their viscoelastic properties and permit a rough evaluation of the constitutive parameters.

2. THREE-ELEMENT MODEL

Polymers can be classified as amorphous (crosslinked or uncrosslinked) and crystalline [1]. Common elastomers fall in the class of crosslinked amorphous polymers while the chord material is generally crystalline. Crosslinking among neighboring molecular chains through the so-called “entanglements” is produced during the manufacturing process and provides the material with a large resistance to the deformation and to the deformation rate.

The viscoelastic behavior of polymers can be detected by creep and stress-relaxation tests. Alternate loading tests give information about the short time phenomena. The deformation is out of phase with the load and a complex elastic modulus can be defined through its two components, storage modulus (real) and loss modulus (imaginary).

It is remarkable that the viscoelasticity of polymers is strongly dependent on the deformation time and we should better define a continuous viscoelastic spectrum in the frequency domain, instead of lumped viscous parameters. The viscoelastic behavior depends also on the temperature in the field above the glass-transition temperature and a correlation can be defined between frequency and temperature dependence [1, 2, 3, 4, 5].

The mechanical response of polymers can be approximately modeled by simple schemes as the Kelvin-Voigt element (spring and dashpot in parallel) or the Maxwell element (spring and dashpot in series). Otherwise, various combinations of Kelvin-Voigt and Maxwell elements give more accurate results. The best fit is obtained by a large number of elements, infinite in the limit, yielding the concept of continuous relaxation or creep spectrum [1, 2, 3].

Figure 1 shows a generalized Maxwell model and the simplified scheme of a belt element that is being considered in the present analysis (Kelvin-Voigt-Maxwell three-parameter element, with asymptotic trend to the rubbery elastic plateau). The constitutive relationship between the belt tension T and the longitudinal elongation ε is

$$T + \tau_T \dot{T} = S(\varepsilon + \tau_\varepsilon \dot{\varepsilon}) \quad (1)$$

where dots indicate derivatives with respect to time, S is the equilibrium stiffness of the belt, τ_T and τ_ε are viscous retardation times, dependent on the duration of the phenomenon. Power belt drives imply deformation times of the order of 10^{-2} s, while static loading on a tension testing machine takes place during several minutes. Very different values of τ_T and τ_ε are to be expected for the two cases.

3. EXPERIMENTS

Figure 2 a shows a schematic representation of the experimental tests. Several commercial V-belts with similar characteristics were wound on two cast-iron pulleys and mounted on a tension testing machine. Other tests were carried out by overturning and winding the V-belts on nylon band pulleys for a flat belt simulation.

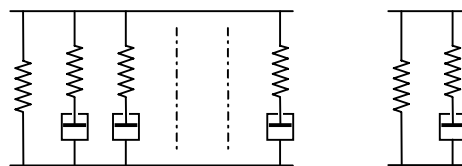


Fig. 1 - Generalized Maxwell model and three-parameter element

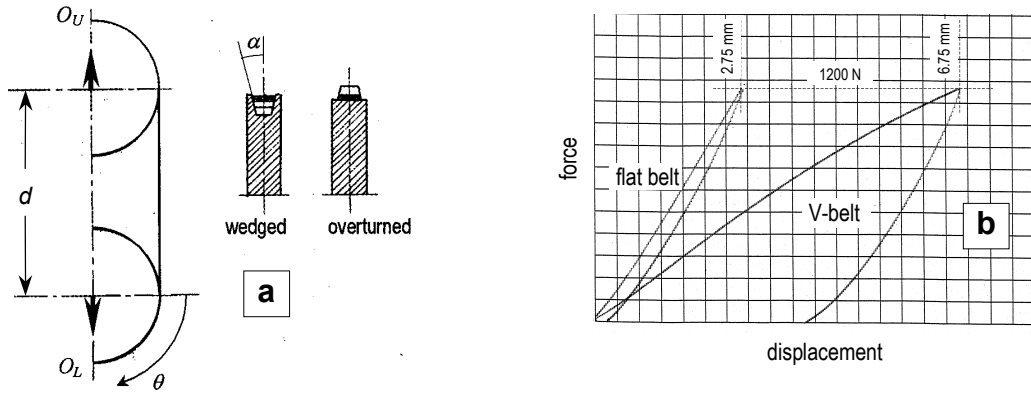


Fig. 2 a, b - Experimental tests, (a): loading scheme, (b): hysteresis cycle

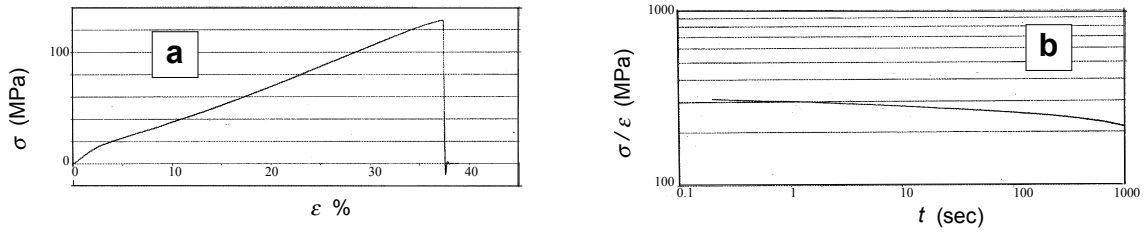


Fig. 3 a, b - Experimental results for a test belt, (a): stress/strain diagram, (b): relaxation (σ = force/cross-section)

The belts were cogged and presented the following characteristics: wedge angle $\alpha = 14^\circ$, length $\cong 750$ mm, transverse width $\cong 15$ mm, thickness $\cong 4$ mm (excluding the cogs), unit length mass $\cong 0.125$ kg/m, winding radii = 39 mm, center distance $\cong 250$ mm. The center distance was increased in all the tests at the constant speeds of 2.5 or 10 mm/min.

The friction coefficient was calculated by dynamometric tests. The values $f \cong 0.3$ and $f \cong 0.12$ were derived for the V- and flat belt test respectively. Notice that actual flat belts exhibit much higher values of f on steel pulleys (e. g. $f = 0.5$).

As an example case, Fig. 2 b shows some experimental load/displacement diagrams for the wedged and overturned arrangements. The hysteresis effect is due to the internal viscosity and to the friction on the pulley walls. V-belts show much higher displacements at the same load levels owing to the elastic wedging into the groove, equivalent to a compliance increase. The slight decrease of the V-belt curve slope on increasing the load is due to the chord creep and to the cross-section contraction.

Moreover, the initial slopes of the unloading diagrams are in practice equal in all the tests, for both wedged and overturned arrangements. At the start of the unloading period in fact, the sliding arc width on the pulley is zero and spreads all over the wrap arc afterwards. Thus, only the free spans begin initially to relax and they are the same for both winding cases.

Relaxation tests were also carried out to calculate the retardation time τ_T of the system. Figure 3 shows a typical stress/strain diagram, up to the belt rupture, and a relaxation plot. The asymptotic equilibrium load $T_\infty = S\varepsilon$ was measured after several minutes of relaxation.

4. LOADING AND UNLOADING ANALYSIS

While loading at a constant deformation rate as in Fig. 2, the elements of a flat belt in contact with the pulley slide outwards from the plane of symmetry $O_U O_L$ and the ends of the wrapped

parts come out from the contact zone. The center distance increase is due to the elongation of the wound parts and the free spans. We can write:

$$d - d_0 = d_0 \varepsilon_{fs} + 2r \int_0^{\pi/2} \varepsilon d\theta \quad (2)$$

where d is the center distance and the subscripts $\dots_{fs}$ and $\dots_0$ refer to the free span and to the zero load condition respectively.

The results for the tests on the grooved pulleys are strongly affected by the belt penetration: $d - d_0 = d_0 \varepsilon_{fs} + 2r \int_0^{\pi/2} (B\varepsilon + x) d\theta$, where $x = -\Delta r / r$ is the dimensionless penetration and the factor $B > 1$ accounts for the transverse bending of the belt cross-section. This case is dealt with in another paper [6], permitting the calculation of the penetration characteristic parameter of the V-belt by comparison with the overturned arrangement.

The two central elements on the plane of symmetry (points O_U and O_L of Fig. 2 a) remain attached to the pulleys while the others slide towards the ends of the contact zone. Then, we must expect the conventional exponential distribution for the belt tension $T = T_{fs}(t) \exp(-f\theta)$, where f is the friction coefficient. Imposing the initial conditions $\varepsilon(0, \theta) = 0$ and $T(0, \theta) = 0$, we have $\varepsilon(t, \theta) = \varepsilon_{fs}(t) \exp(-f\theta)$, where $\varepsilon_{fs}(t)$ is solution of Eq. (1) in the time domain, i. e. replacing $T(t, \theta)$ with $T_{fs}(t)$.

For a constant deformation rate v , Eq. (2) yields $d - d_0 = vt = b \varepsilon_{fs}(t)$ where $b = d_0 + 2r[1 - \exp(-f\pi/2)]/f$, whence we deduce that

$$\varepsilon(t, \theta) = vt \exp(-f\theta) / b \quad (3)$$

Now, we turn to Eq. (1) and solve for $T_{fs}(t)$. Then

$$T(t, \theta) = Sv \{t + (\tau_\varepsilon - \tau_T)[1 - \exp(-t/\tau_\varepsilon)]\} \exp(-f\theta) / b \quad (4)$$

Suppose that the unloading starts immediately after the end of the loading period. The belt parts that were drawn out previously must return into the wrap arc and the friction forces are now counter-directed. Nevertheless, we cannot presume sudden sliding on the whole wrap arcs towards the central points O_U and O_L , as the belt tension decreases towards the inside at the initial moment by Eq. (4). Thus, we must expect each half arc of contact to be divided in two parts, a sliding arc whose width increases from zero to $\pi/2$, and an adhesion arc, where ε keeps the values reached at the end of the loading period t_x (see Eq. (3)):

$$\varepsilon(\theta) = vt_x \exp(-f\theta) / b \quad (5)$$

Let us measure now the time t from the beginning of the unloading period. The tension increases with θ according to the Euler law along the sliding arc, $T(t, \theta) = T_{fs}(t) \exp(f\theta)$, while it relaxes according to the relation $T(t, \theta) = Sv \{t_x + (\tau_\varepsilon - \tau_T)[1 - \exp(-t_x/\tau_T)]\} \exp(-f\theta) / b$ along the adhesion zone by Eqs. (1), (4) and (5). The moving boundary between the two arcs is located where the two expressions give the same T (tension continuity):

$$Sv\{t_x + (\tau_\varepsilon - \tau_T)[1 - \exp(-t_x/\tau_T)]\exp(-t/\tau_T)\}/b = T_{fs}(t) \exp(2f\theta_b) \quad (6)$$

where $\theta_b(t)$ is the angular coordinate of the boundary.

The elongation can be obtained along the sliding arc by solving Eq. (1) for ε formally:

$$\varepsilon(t, \theta) = \left[\varepsilon_{fs}(t) - \varepsilon_{fs}(t_b) \exp\left(-\frac{t-t_b}{\tau_\varepsilon}\right) \right] \exp(f\theta) + \varepsilon_{fs}(0) \exp(-f\theta) \exp\left(-\frac{t-t_b}{\tau_\varepsilon}\right) \quad (7)$$

where $t_b = t_b(\theta)$ is the inverse function of $\theta_b(t)$, $t_b(0) = 0$ and $\varepsilon_{fs}(0) = v t_x / b$. This solution holds for $t > t_b$ and connects with (5) for $t = t_b(\theta)$ (elongation continuity).

At last, assuming that the deformation speed is $-v$ in the unloading period, Eq. (2) gives:

$$d - d_0 = v(t_x - t) = d_0 \varepsilon_{fs}(t) + 2r \left[\int_0^{\theta_b} \varepsilon_{\text{Eq.(7)}} d\theta + \int_{\theta_b}^{\pi/2} \varepsilon_{\text{Eq.(5)}} d\theta \right] \quad (8)$$

while Eq. (1) implies:

$$T_{fs} + \tau_T \dot{T}_{fs} = S(\varepsilon_{fs} + \tau_\varepsilon \dot{\varepsilon}_{fs}) \quad (9)$$

We have thus three equations in total (6, 8, 9), in the three unknowns $T_{fs}(t)$, $\varepsilon_{fs}(t)$ and $\theta_b(t)$. Owing to the non-linear nature of the problem and to the unknown moving boundary, the solution cannot be obtained but by some iterative numerical procedure, e. g. dividing the integration domain $0 < \theta < \pi/2$ in N equal intervals, $\theta_{i+1} - \theta_i$. A tentative function $t_b(\theta_i)$ is assumed and Eq. (6) is easily solved for $T_{fs}(t_i)$, while Eq. (8) becomes an integral equation for the unknown function $\varepsilon_{fs}(t_i)$, to be solved apart by successive approximations. The results are introduced into Eq. (9) and the values $t_b(\theta_i)$ are corrected for a better approximation. This process is then repeated until the desired convergence is attained.

After the sliding arc has spread as far as the whole contact, the procedure becomes simpler, because Eqs. (7) and (8) yield $\varepsilon_{fs}(t)$, putting $\theta_b = \pi/2$, $t_b = t_b(\pi/2)$ and replacing $\varepsilon_{fs}(0) \exp(-f\theta)$ with $\varepsilon[t_b(\pi/2), \theta]$ into Eq. (7). Then, Eq. (9) can be solved for $T_{fs}(t)$.

By a proper choice of the time constants τ_T and τ_ε , a very good fit of the theoretical diagram and the load-unload experimental curve can be achieved.

It is interesting that the free span elongation speed is $\dot{\varepsilon}_{fs} = -v/d_0$ at the unloading start for both the V- and flat belt. This property can be deduced by differentiation of Eq. (8) according with the Leibnitz rule, minding that $\theta_b(0) = 0$ and $\varepsilon_{\text{Eq.(5)}}$ does not depend on t . Thus, the initial slope of the unloading diagram is nearly the same for the two belt arrangements, as confirmed by the experimental plots.

Notice that the solution can be obtained in closed form in the pure elastic case ($\tau_T = \tau_\varepsilon = 0$), where we have $T = S\varepsilon$ at any time. Equations (5) to (8) lead to the result $\exp(-f\theta_b) = \{[(fd_0)^2 - fbt_b(fd_0 - 2r) / t_x]^{1/2} - 2r\} / (fd_0 - 2r)$ as long as $\theta_b < \pi/2$, and then $\varepsilon_{fs}(t)$ and $T_{fs}(t)$ can be easily calculated. Ideal elasticity requires that the belt tension and the elongation should vanish simultaneously at the end of the unloading period and the hysteresis cycle should close

at the origin of the diagram (T, d). Nevertheless, this is in contrast with the experimental results and gives a further proof of the viscoelastic effect during the static loading test.

Relaxation tests after a sudden elongation of a piece of belt were also carried out on the testing machine (see Fig. 3 b). Basing on the time law $T - T_\infty = (T_x - T_\infty) \exp(-t/\tau_T)$, where $T_\infty = S\varepsilon$, useful additional information can be obtained about the time constant τ_T .

By interpretation of the experimental results in accordance with the above theory, a realistic estimation of τ_T and τ_ε can be derived, $\tau_T \cong 27$ s, $\tau_\varepsilon \cong 40$ s, for nearly all the tested belts. The equilibrium stiffness was $S \cong 60000$ N.

Of course, the short time phenomena occurring in a power drive are governed by different time constants. Reasonable hypotheses about the order of magnitude of the short time constants can refer to some experimental results for variable frequency and/or temperature [5] (increasing the temperature has the same effect on the viscoelastic properties as decreasing the frequency). A crude extrapolation yields the approximate relationship $\tau_T \cong 0.2/\omega$ s, $\tau_\varepsilon \cong 0.3/\omega$ s, where ω is the angular speed of the pulley.

5. VISCOELASTIC BELT MECHANICS

Let us assume a flat belt drive in steady working. Considering the material response from the Eulerian point of view, the constitutive equation Eq. (1) must be written as

$$\begin{aligned} T + \omega\tau_T T' &= S(\varepsilon + \omega\tau_\varepsilon \varepsilon') & (\text{along the arc of contact}) \\ T + v\tau_T \frac{dT}{ds} &= S\left(\varepsilon + v\tau_\varepsilon \frac{d\varepsilon}{ds}\right) & (\text{along the free span}) \end{aligned} \quad (10 \text{ a, b})$$

where primes indicate derivatives with respect to the angular coordinate θ , ω is the angular velocity of the pulley, v is the belt velocity and s is the linear coordinate along the free span.

Neglecting the momentum flux qv^2 , we must have $T' = \pm fT$ along a sliding arc and $\varepsilon' = 0$ along an adhesion arc. Therefore, considering as a known quantity the left hand of (10 a) in the first case and the right hand in the second case, it is possible to solve with respect to the local unknown variable (ε or T). Moreover, we have $dT/ds = 0$ in Eq. (10 b), as no external forces are applied along the free span, and it is possible to solve for $\varepsilon(s)$.

An essential difference with the conventional Euler-Grashof analysis is that sliding zones can be present at the entrance of the arc of contact in some particular conditions. As a matter of fact, the tension of a viscoelastic belt is variable along the adhesion zone and, should the derivative T' become there higher than $f_{st}T$ in modulus (f_{st} : coefficient of static friction, which is here assumed $=f$), the pulley walls could not sustain the adhesion contact and sliding would occur. The adhesion arc will then begin where $|T'| = fT$.

The mass conservation condition can be written in the usual form $v/(1 + \varepsilon) = v_P/(1 + \varepsilon_P)$, where P is a reference point somewhere along the belt trajectory. Then, if $T' < 0$ at the boundary between an entrance sliding arc and an adhesion arc, the belt speed is certainly increasing in the whole seating zone upstream, and vice versa. In fact, Eq. (10 a) gives $T(1 \pm f\omega\tau_\varepsilon) = S\varepsilon$ at this point (where $\varepsilon' = 0$), and $T(1 \pm f\omega\tau_\varepsilon) = S(\varepsilon + \omega\tau_T \varepsilon')$ a little bit upstream, and thus an increase (decrease) of T toward upstream must be accompanied by an increase (decrease) of ε' and then by a decrease (increase) of ε and of the belt speed.

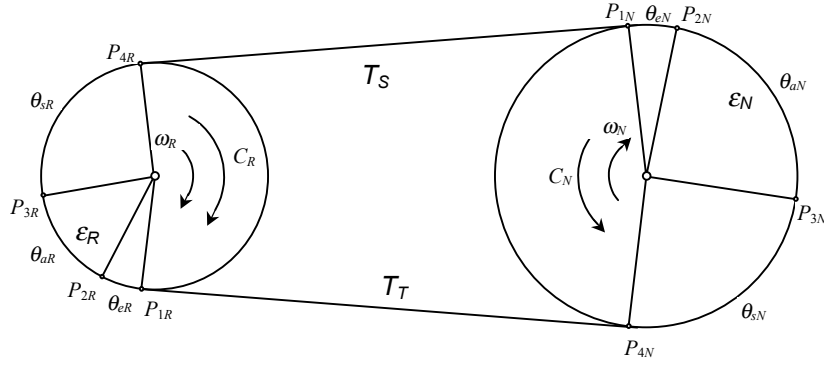


Fig. 4 - Belt drive regions (P_1P_2 = entrance sliding arc, P_2P_3 = adhesion arc, P_3P_4 = main sliding arc)

If the entrance sliding arcs are present, the belt speed cannot but increase at the entrance of a driver pulley and decrease for a driven pulley, otherwise the tight free span would run faster than the driver pulley or the slack free span would run slower than the driven pulley.

Moreover, if the belt inertia cannot be neglected (qv^2), one may introduce the “dynamic” tension $T - qv^2$ and elongation $\varepsilon - qv^2/S$, and minding that $qv = q_P v_P$ due to the mass conservation and $(qv^2)' \cong 0$, all the equations of the present section remain unchanged, replacing the “static” variables with their “dynamic” counterparts.

Divide the belt length in various parts as in Fig. 4, indicate with T_T and T_S the belt forces in the tight and slack spans and with ε_R and ε_N the longitudinal elongation in the adhesion regions of the driver and driven pulleys (the subscripts ... R and ... N will refer to the driver and driven pulleys in the following).

Solving the belt equations for each partial domain and connecting the solutions to each other, the following results can be obtained in a compact form for the driver and driven pulley. Whenever the symbol $\langle \dots | \dots \rangle$ shows up, the left and right signs refer to the driver and driven pulleys respectively. The angular coordinate θ is always measured from the initial point of each examined region.

- Entrance sliding arc $\langle P_{1R}P_{2R} | P_{1N}P_{2N} \rangle$, if present

$$T = T_{\langle T|S \rangle} e^{\langle -|+ \rangle f \theta} \quad (11 \text{ a, b})$$

$$\varepsilon = \frac{T_{\langle T|S \rangle} \tau_T}{S \tau_\varepsilon} \left(\frac{p_{T\langle R|N \rangle} \langle -|+ \rangle 1}{p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1} \right) e^{\langle -|+ \rangle f \theta_{e\langle R|N \rangle}} \left[e^{\langle -|+ \rangle f (\theta - \theta_{e\langle R|N \rangle})} - e^{-f p_{\varepsilon\langle R|N \rangle} (\theta - \theta_{e\langle R|N \rangle})} \right] + \varepsilon_{\langle R|N \rangle} e^{-f p_{\varepsilon\langle R|N \rangle} (\theta - \theta_{e\langle R|N \rangle})} \quad (12 \text{ a, b})$$

where $\theta_{e\langle R|N \rangle}$ is the angular width of the entrance sliding arc, and we have put $p_{T\langle R|N \rangle} = 1 / (f \tau_T \omega_{\langle R|N \rangle})$ and $p_{\varepsilon\langle R|N \rangle} = 1 / (f \tau_\varepsilon \omega_{\langle R|N \rangle})$.

- Adhesion arc $\langle P_{2R}P_{3R} | P_{2N}P_{3N} \rangle$

$$T = S \varepsilon_{\langle R|N \rangle} + \left(T_{\langle T|S \rangle} e^{\langle -|+ \rangle f \theta_{e\langle R|N \rangle}} - S \varepsilon_{\langle R|N \rangle} \right) e^{-f p_{T\langle R|N \rangle} \theta} \quad (13 \text{ a, b})$$

$$\varepsilon = \varepsilon_{\langle R|N \rangle} \quad (14 \text{ a, b})$$

- Sliding arc $\langle P_{3R}P_{4R} | P_{3N}P_{4N} \rangle$

$$T = \left[S\varepsilon_{\langle R|N \rangle} + \left(T_{\langle T|S \rangle} e^{\langle -|+ \rangle f\theta_{e\langle R|N \rangle}} - S\varepsilon_{\langle R|N \rangle} \right) e^{-fp_{T\langle R|N \rangle}\theta_{a\langle R|N \rangle}} \right] e^{\langle -|+ \rangle f\theta} \quad (15 \text{ a, b})$$

$$\varepsilon = \frac{\tau_T}{\tau_\varepsilon} \left(\frac{p_{T\langle R|N \rangle} \langle -|+ \rangle 1}{p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1} \right) \left[\varepsilon_{\langle R|N \rangle} + \left(\frac{T_{\langle T|S \rangle} e^{\langle -|+ \rangle f\theta_{e\langle R|N \rangle}}}{S} - \varepsilon_{\langle R|N \rangle} \right) e^{-fp_{T\langle R|N \rangle}\theta_{a\langle R|N \rangle}} \right] \times \quad (16 \text{ a, b})$$

$$\times \left(e^{\langle -|+ \rangle f\theta} - e^{-fp_{\varepsilon\langle R|N \rangle}\theta} \right) + \varepsilon_{\langle R|N \rangle} e^{-fp_{\varepsilon\langle R|N \rangle}\theta}$$

where $\theta_{a\langle R|N \rangle}$ is the angular width of the adhesion arc.

- Free span downstream $\langle P_{4R}P_{1N} | P_{4N}P_{1R} \rangle$

$$T = T_{\langle S|T \rangle} = \left[S\varepsilon_{\langle R|N \rangle} + \left(T_{\langle T|S \rangle} e^{\langle -|+ \rangle f\theta_{e\langle R|N \rangle}} - S\varepsilon_{\langle R|N \rangle} \right) e^{-fp_{T\langle R|N \rangle}\theta_{a\langle R|N \rangle}} \right] e^{\langle -|+ \rangle f\theta_{s\langle R|N \rangle}} \quad (17 \text{ a, b})$$

$$\varepsilon = \left[\varepsilon_{\langle R|N \rangle} + \left(\frac{T_{\langle T|S \rangle} e^{\langle -|+ \rangle f\theta_{e\langle R|N \rangle}}}{S} - \varepsilon_{\langle R|N \rangle} \right) e^{-fp_{T\langle R|N \rangle}\theta_{a\langle R|N \rangle}} \right] \left\{ e^{\langle -|+ \rangle f\theta_{s\langle R|N \rangle}} + \right.$$

$$+ \left[\langle +|- \rangle \left(\frac{1 - \tau_T/\tau_\varepsilon}{p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1} \right) e^{\langle -|+ \rangle f\theta_{s\langle R|N \rangle}} - \frac{\tau_T}{\tau_\varepsilon} \left(\frac{p_{T\langle R|N \rangle} \langle -|+ \rangle 1}{p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1} \right) e^{-fp_{\varepsilon\langle R|N \rangle}\theta_{s\langle R|N \rangle}} \right] e^{-\frac{s}{\tau_\varepsilon v}} \Bigg\} + \quad (18 \text{ a, b})$$

$$+ \varepsilon_{\langle R|N \rangle} e^{-\frac{s}{\tau_\varepsilon v} - fp_{\varepsilon\langle R|N \rangle}\theta_{s\langle R|N \rangle}}$$

where $\theta_{s\langle R|N \rangle}$ is the width of the main sliding arc.

Putting $\theta = 0$ into Eqs. (12 a, b) and $s = s_{fs}$ into Eqs. (18 b, a) (s_{fs} = free length), equating the two expressions of the elongation $\varepsilon_{1\langle R|N \rangle}$ and accounting for Eq. (17 a, b), we get

$$\frac{T_{\langle T|S \rangle} \tau_T}{\tau_\varepsilon} \left(\frac{p_{T\langle R|N \rangle} \langle -|+ \rangle 1}{p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1} \right) \left[1 - e^{f\theta_{e\langle R|N \rangle} (p_{\varepsilon\langle R|N \rangle} \langle -|+ \rangle 1)} \right] + S\varepsilon_{\langle R|N \rangle} e^{fp_{\varepsilon\langle R|N \rangle}\theta_{e\langle R|N \rangle}} =$$

$$= T_{\langle T|S \rangle} \left\{ 1 + \left[\langle -|+ \rangle \left(\frac{1 - \tau_T/\tau_\varepsilon}{p_{\varepsilon\langle N|R \rangle} \langle +|- \rangle 1} \right) - \frac{\tau_T}{\tau_\varepsilon} \left(\frac{p_{T\langle N|R \rangle} \langle +|- \rangle 1}{p_{\varepsilon\langle N|R \rangle} \langle +|- \rangle 1} \right) e^{-f(p_{\varepsilon\langle N|R \rangle} \langle +|- \rangle 1)\theta_{s\langle N|R \rangle}} \right] \times \right.$$

$$\times e^{-\frac{s_{fs}}{\tau_\varepsilon v}} \Bigg\} + S\varepsilon_{\langle N|R \rangle} e^{-\frac{s_{fs}}{\tau_\varepsilon v} - fp_{\varepsilon\langle N|R \rangle}\theta_{s\langle N|R \rangle}} \quad (19 \text{ a, b})$$

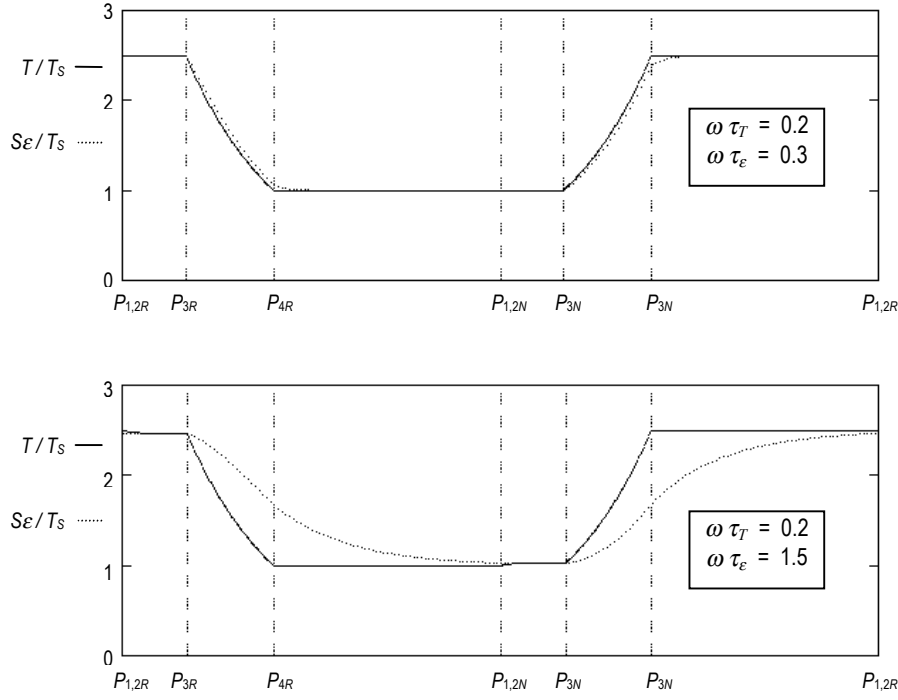


Fig. 5 - Force and elongation diagrams ($r = 40$ mm, $d_0 = 250$ mm, $f = 0.5$, $T_T / T_S = 2.5$)

Two further relationships derive from the condition $|T' / T| = f$ at the end of the entrance sliding arcs. Equations (13 a, b) yield

$$\exp(-f\theta_{eR}) = \frac{S\epsilon_R p_{TR}}{T_T(p_{TR} - 1)} \quad \exp(f\theta_{eN}) = \frac{S\epsilon_N p_{TN}}{T_S(p_{TN} + 1)} \quad (20 \text{ a, b})$$

Collecting Eqs. (17 a, 17 b, 19 a, 19 b, 20 a, 20 b), we have six equations in total for the six unknowns θ_{eR} , θ_{sR} , θ_{eN} , θ_{sN} , ϵ_R and ϵ_N . This system is non-linear and moreover the constraints $\theta_{eR} \geq 0$, $\theta_{eN} \geq 0$ must be controlled. Only solutions by numerical procedure can be attempted. Here, the steepest-descent method was applied to the minimization of the criterion function $\Phi = 0.5 \sum (\Phi_i^2)$, where the Φ_i 's are residual errors of the system equations. The gradient vector was calculated numerically and a dicotomic convergence process was used at each step. Whenever a negative value resulted for one of the seating arc widths, this value was changed to zero.

Figure 5 shows the tension and elongation distribution for some values of τ_T and τ_ϵ . The first case refers to the estimation of the previous section and is nearly equivalent to the pure elastic case. The second case refers to a higher value of τ_ϵ and exhibits a remarkable viscoelastic behavior. These two cases do not exhibit seating sliding arcs, which may show up on the contrary for values of τ_T much smaller than τ_ϵ . It is interesting that equal creep and relaxation times ($\tau_T = \tau_\epsilon$) lead to an elastic-like behavior, since Eq. (1) implies the proportionality of T and ϵ . Therefore, the viscoelastic effect becomes important when there is a remarkable difference between the two time constants.

The numerical results indicate that a relevant reduction of the sliding speed $v_{sliding}$, proportional to $\epsilon - \epsilon_{\langle R|N \rangle}$, can be obtained by highly viscoelastic belt chords, as the viscous resistance retards the decrease (increase) of the belt velocity along the main sliding arc of the

driver (driven) pulley. This effect may yield some advantage as regards the wear reduction, the belt durability and, though to a very small amount, also the efficiency in spite of the additional viscous loss. By choosing the chord material accurately or embedding the chord into a thin layer of elastomer with a high loss factor [3], the chord viscoelasticity can be substantially upraised.

On the contrary, any viscous effect is undesired in the rubber body of the belt because of the flexural hysteresis loss.

6. CONCLUSION

An extensive insight into the viscoelastic behavior of the traction belts is presented and the advantages achievable in terms of wear resistance by exploiting such a physical property are outlined. Experimental evidence of the belt viscous response is also shown.

It is remarkable that the viscoelastic resistance of polymers can be raised as much as desired, within certain limits, by a proper manufacturing process, which opens interesting prospects in the technology of rubber belts and in the application field.

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